

$$\textcircled{53.} \quad \begin{aligned} a^2 &= b^2 + c^2 \\ l^2 &= b^2 + 10^2 \\ l &= 11.66 \text{ cm} \end{aligned}$$

$$\text{a)} \quad \begin{array}{c} \text{Base: } 24 \times 12 \text{ cm} \\ \text{Altura: } 6 \text{ cm} \\ \text{Área lateral: } A_L = 24 \cdot 6 = 144 \text{ cm}^2 \\ \text{Área de la base: } A_B = \frac{\pi \cdot 12^2}{4} = 113.09 \text{ cm}^2 \\ \text{Área total: } A_T = A_L + 2A_B = 144 + 2 \cdot 113.09 = 360.18 \text{ cm}^2 \\ \text{Volumen: } V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \cdot 12^2 \cdot 6 = 2985 \text{ cm}^3 \end{array}$$

Tiene más volumen la esfera.

$$\text{b)} \quad \begin{array}{c} \text{Base: } 24 \times 16 \text{ cm} \\ \text{Altura: } 12 \text{ cm} \\ \text{Área lateral: } A_L = 24 \cdot 12 = 288 \text{ cm}^2 \\ \text{Área de la base: } A_B = \frac{\pi \cdot 12^2}{4} = 113.09 \text{ cm}^2 \\ \text{Área total: } A_T = A_L + 2A_B = 288 + 2 \cdot 113.09 = 514.18 \text{ cm}^2 \\ \text{Volumen: } V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \cdot 12^2 \cdot 12 = 565.2 \text{ cm}^3 \end{array}$$

$$\text{81.} \quad \begin{aligned} A_{\text{cuadrado}} &= 6a^2 = 216 \Rightarrow a = 6 \text{ cm} \Rightarrow \sqrt{a^2} = \sqrt{6^2} = 6 \text{ cm} \\ A_{\text{esfera}} &= 4\pi r^2 = 216 \Rightarrow r = \sqrt{\frac{216}{4\pi}} = 4.15 \text{ cm} \\ V_{\text{esfera}} &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 4.15^3 = 2985 \text{ cm}^3 \end{aligned}$$

35 ESO

$$\text{85.} \quad \begin{array}{c} \text{Base: } 80 \text{ cm} \\ \text{Altura: } 20 \text{ cm} \\ \text{Área total: } A_T = \frac{P_b \cdot a}{2} = \frac{80 \cdot 20}{2} = 800 \Rightarrow a = \frac{80 \cdot 2}{32} = 5 \text{ cm} \end{array}$$

$$\text{56.} \quad \begin{array}{c} \text{Base: } 24 \times 16 \text{ cm} \\ \text{Altura: } 12 \text{ cm} \\ \text{Área lateral: } A_L = 24 \cdot 12 = 288 \text{ cm}^2 \\ \text{Área de la base: } A_B = \frac{P_b \cdot a}{2} + \frac{D \cdot d}{2} = \frac{24 \cdot 16}{2} + \frac{20 \cdot 12}{2} = 2318 + 120 = 2438 \text{ cm}^2 \\ a = \sqrt{24^2 - 5^2} = 23.28 \text{ cm} \end{array}$$

$$\text{56.} \quad \begin{array}{c} \text{Base: } 16 \times 18 \text{ cm} \\ \text{Altura: } 30 \text{ cm} \\ \text{Área lateral: } A_L = 16 \cdot 30 = 480 \text{ cm}^2 \\ \text{Área de la base: } A_B = \frac{P_b \cdot a}{2} = \frac{16 \cdot 18}{2} = 144 \text{ cm}^2 \\ a = \sqrt{16^2 + 12^2} = \sqrt{400} = 20 \text{ cm} \end{array}$$

$$\text{60.} \quad \begin{array}{c} \text{Base: } 10 \times 8 \text{ cm} \\ \text{Altura: } 10 \text{ cm} \\ \text{Área lateral: } A_L = 10 \cdot 8 = 80 \text{ cm}^2 \\ \text{Área de la base: } A_B = \frac{P_b \cdot a}{2} = \frac{10 \cdot 8}{2} = 40 \text{ cm}^2 \\ \text{Volumen: } V = \frac{1}{3} A_B \cdot h = \frac{1}{3} \cdot 40 \cdot 10 = \frac{400}{3} \text{ cm}^3 \end{array}$$

$$\text{66.} \quad \begin{array}{c} \text{Base: } 10 \times 8 \text{ cm} \\ \text{Altura: } 10 \text{ cm} \\ \text{Área lateral: } A_L = 10 \cdot 8 = 80 \text{ cm}^2 \\ \text{Área de la base: } A_B = \frac{P_b \cdot a}{2} = \frac{10 \cdot 8}{2} = 40 \text{ cm}^2 \\ \text{Volumen: } V = \frac{1}{3} A_B \cdot h = \frac{1}{3} \cdot 40 \cdot 10 = \frac{400}{3} \text{ cm}^3 \end{array}$$

$$\text{56.} \quad \begin{array}{c} \text{Base: } 16 \times 18 \text{ cm} \\ \text{Altura: } 30 \text{ cm} \\ \text{Área lateral: } A_L = 16 \cdot 30 = 480 \text{ cm}^2 \\ \text{Área de la base: } A_B = \frac{P_b \cdot a}{2} = \frac{16 \cdot 18}{2} = 144 \text{ cm}^2 \\ a = \sqrt{16^2 + 12^2} = \sqrt{400} = 20 \text{ cm} \end{array}$$

$$A = A_L + A_B + A_B = \pi(r+l)g + \pi R^2 + \pi r^2 = \pi(30+18) \cdot 20 + \pi \cdot 30^2 + \pi \cdot 18^2 = 2184\pi = \boxed{6861.24 \text{ cm}^2}$$

$$\frac{h}{h+16} = \frac{18}{30} \Rightarrow 30h = 18h + 288 \Rightarrow h = 24 \text{ cm}$$

$$\text{b)} \quad \begin{array}{c} \text{V: } V = V_{\text{cilindro}} + V_{\text{pirámide}} = abc + \frac{A_B \cdot h}{3} = 4 \cdot 2 \cdot 2 + \frac{2 \cdot 2}{3} = 18.67 \text{ cm}^3 \\ V = V_{\text{cilindro}} + V_{\text{pirámide}} = \pi r^2 h + \frac{\pi r^2 h}{3} = \pi \cdot 3^2 \cdot 4 + \frac{\pi \cdot 3^2 \cdot 4}{3} = 48\pi = \boxed{150.8 \text{ cm}^3} \end{array}$$

$$\text{c)} \quad V = V_{\text{cilindro}} - V_{\text{pirámide}} = \pi r^2 h - \frac{\pi r^2 h}{3} = \pi \cdot 4^2 \cdot 8 - \frac{\pi \cdot 4^2 \cdot 4}{3} = \boxed{335.1 \text{ cm}^3}$$

$$\text{d)} \quad V = V_{\text{cilindro}} - g \cdot V_{\text{pirámide}} = 9^3 - 8 \cdot 3^3 = \boxed{513 \text{ cm}^3}$$

$$\text{e)} \quad V = V_{\text{cilindro}} + V_{\text{pirámide}} = \frac{1}{2}(\pi \cdot 15^2 \cdot 5 + \pi \cdot 15^2 \cdot 5) = 75\pi = \boxed{2356 \text{ cm}^3}$$

$$\text{f)} \quad V = V_{\text{cilindro}} - V_{\text{pirámide}} = 6^3 - \frac{6^2 \cdot 4}{3} = \boxed{168 \text{ cm}^3}$$

$$\begin{aligned} V &= V_{\text{cilindro}} - V_{\text{pirámide}} = 6^3 - \frac{6^2 \cdot 4}{3} = 216 - 48 = 168 \text{ cm}^3 \\ &= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (30 \cdot 40 - 18 \cdot 24) = 960\pi = \boxed{30168 \text{ cm}^3} \\ &= \frac{1}{3}\pi (1200 - 432) = \boxed{29556 \text{ cm}^3} \end{aligned}$$

$$g) V = V_{\text{cubo}} - V_{\text{pirámide}} = 8^3 - \frac{1}{3} \cdot \frac{4 \cdot 4}{2} \cdot 4 = \boxed{501.33 \text{ cm}^3}$$

(Pirámide triangular de base un triángulo rectángulo de catetos de 4cm)

$$h) V = V_{\text{pirámide}} + V_{\text{cilindro}} = \frac{4}{3} \pi r^3 + \pi r^2 h = \frac{4}{3} \pi \cdot 6^3 + \pi \cdot 6^2 \cdot 7 = \boxed{5407 \pi = 16961.46 \text{ cm}^3}$$

$$(83) \quad a) \begin{array}{c} \triangle \\ \text{base: } 12 \\ \text{altura: } 12+x \\ \text{hipotenusa: } 12+x \\ \text{cateto: } 6 \end{array} \quad \frac{12}{12+x} = \frac{3}{6} \Rightarrow 72 = 36 + 3x \Rightarrow x = 12 \Rightarrow H = 24 \text{ cm}$$

$$\sqrt{\frac{1}{3} A_B + H - \frac{1}{3} A_B \cdot h} = \frac{1}{3} (12 \cdot 24 - 6 \cdot 12) = \boxed{1008 \text{ cm}^3}$$

$$b) \begin{array}{c} \triangle \\ \text{base: } 6 \\ \text{altura: } 6 \\ \text{hipotenusa: } 10 \\ \text{cateto: } 8 \end{array} \quad \sqrt{\frac{1}{3} h^2 + 2^2} = 6^2 \quad \sqrt{\frac{1}{3} \pi (R^2 H - r^2 h)} = \frac{1}{3} \pi (4^2 \cdot 132 - 2^2 \cdot 566) = \boxed{1651.96 \text{ cm}^3}$$

$$H = 1132 \text{ cm}$$

$$c) 5 = \sqrt{a^2 + a^2 + a^2} \Rightarrow 5 = \sqrt{3a^2} \Rightarrow a = \sqrt{\frac{5}{3}} = 2.9 \text{ cm}$$

$$\sqrt{-a^3} = \sqrt[2]{a^3} = \boxed{24.06 \text{ cm}^3}$$

$$d) \begin{array}{c} \triangle \\ \text{base: } 8 \\ \text{altura: } 6 \\ \text{hipotenusa: } 10 \\ \text{cateto: } 6 \end{array} \quad a^2 = b^2 + c^2$$

$$a^2 = 64 + \frac{64}{4} \Rightarrow a^2 - \frac{a^2}{4} = 64 \Rightarrow \frac{3a^2}{4} = 64 \Rightarrow a = \frac{8\sqrt{3}}{3} \Rightarrow a = \frac{16}{\sqrt{3}}$$

$$h = \sqrt{x^2 - \left(\frac{8}{3}\right)^2} = 7.54 \text{ cm}$$

$$A = \frac{1}{3} A_B \cdot h = \frac{1}{3} \cdot \frac{924 \cdot 8}{2} \cdot 7.54 = \boxed{921.87 \text{ cm}^2}$$

$$(99) \quad \text{Nuevo envase: } A_B = 66 \text{ cm}^2 \rightarrow A'_B = 0.9 \cdot 66 = 59.4 \text{ cm}^2$$

$$h = 15 \rightarrow h = 11.15 = 16.5 \text{ cm}$$

$$a) V_{\text{antiguo}} = 11.6 \cdot 15 = 990 \text{ cm}^3$$

Es menor

$$b) \text{No, pagaría lo mismo por metros}$$

$$c) \begin{array}{l} \text{Ahora: } (99000 : 0.98) \cdot 140 \text{ €} = 141428.57 \text{ €} \\ \text{Antes: } (99000 : 0.99) \cdot 140 \text{ €} = 140000 \text{ €} \end{array}$$

$$(93) \quad \begin{array}{c} \triangle \\ \text{base: } 2 \\ \text{altura: } 1 \\ \text{hipotenusa: } \sqrt{2+1^2} \\ \text{cateto: } 1 \end{array} \quad h = \sqrt{2+1^2} \quad h = \sqrt{3} \quad A_{\text{base}} = \frac{b \cdot h}{2} = \frac{2 \cdot \sqrt{3}}{2} = \sqrt{3} =$$

Atrapezoido = 4. A_{base} = 4\sqrt{3} =

$$(94) \quad \begin{array}{c} \triangle \\ \text{base: } 4 \\ \text{altura: } 2 \\ \text{hipotenusa: } \sqrt{4+2^2} \\ \text{cateto: } 2 \end{array} \quad h = \sqrt{4+2^2} \quad h = \sqrt{20} = 2\sqrt{5} \quad A_{\text{base}} = \frac{b \cdot h}{2} = \frac{4 \cdot 2\sqrt{5}}{2} = 4\sqrt{5} =$$

Atrapezoido = 8. A_{base} = 8 \cdot 4\sqrt{5} = 32\sqrt{5} =

$$(95) \quad \begin{array}{c} \triangle \\ \text{base: } 6 \\ \text{altura: } 6 \\ \text{hipotenusa: } \sqrt{6+6^2} \\ \text{cateto: } 6 \end{array} \quad h = \sqrt{6+6^2} \quad h = \sqrt{72} = 6\sqrt{3} \quad A_{\text{base}} = \frac{b \cdot h}{2} = \frac{6 \cdot 6\sqrt{3}}{2} = 18\sqrt{3} =$$

Atrapezoido = 20. A_{base} = 20 \cdot 6\sqrt{3} = 120\sqrt{3} =

$$(96) \quad \begin{array}{c} \triangle \\ \text{base: } l \\ \text{altura: } \frac{l}{2} \\ \text{hipotenusa: } \sqrt{l^2 + \left(\frac{l}{2}\right)^2} \end{array} \quad h = \sqrt{l^2 + \left(\frac{l}{2}\right)^2} \Rightarrow l^2 = h^2 + \frac{l^2}{4} \Rightarrow l^2 - \frac{l^2}{4} = h^2 \Rightarrow h = \frac{3l}{4} =$$

$$a) A_{\Delta} = \frac{16\sqrt{3}}{4} = 4\sqrt{3}$$

$$\frac{l^2\sqrt{3}}{4} = 4\sqrt{3} \Rightarrow l^2 = 16 \Rightarrow \boxed{l = 4 \text{ cm}}$$

$$b) A_{\Delta} = \sqrt{3} = \frac{l^2\sqrt{3}}{4} \Rightarrow l^2 = 4 \Rightarrow \boxed{l = 2 \text{ cm}}$$

$$c) A_{\Delta} = \frac{18\sqrt{3}}{8} = \frac{9\sqrt{3}}{4} \Rightarrow \frac{9\sqrt{3}}{4} = \frac{9\sqrt{3}}{4} \Rightarrow l = 9 \Rightarrow \boxed{l = 3 \text{ cm}}$$

$$(97) \quad \begin{array}{c} \text{rectangular} \\ \text{base: } 1 \\ \text{altura: } 2 \\ \text{profundidad: } 1 \end{array} \quad \sqrt{2^2 + (4x)^2 + (2x)^2} = \sqrt{21x^2} \Rightarrow x = 1 \text{ cm}$$

$$A = P_B \cdot h + 2 \cdot A_S = 12 \cdot 1 + 2 \cdot 4 \cdot 2 = \boxed{28 \text{ cm}^2}$$

$$(90) \quad V_{\text{cubo}} = a^3 = 125 \Rightarrow a = 5 \text{ cm} \Rightarrow A_{\text{superf}} = 6a^2 = 6 \cdot 5^2 = 150 \text{ cm}^2$$

$$\begin{aligned} \text{Verifica: } & \frac{4}{3} \pi r^3 = 125 \Rightarrow r = \sqrt[3]{\frac{125 \cdot 3}{4\pi}} = 3.1 \text{ cm} \\ \text{A superf: } & 4\pi r^2 = 4\pi \cdot 3^2 = 120\pi \text{ cm}^2 \end{aligned}$$

Se necesita menos material con una espesa

$$91) \text{ Vespresa} = \frac{4}{3}\pi r^3 = 24416640 \text{ dm}^3 \Rightarrow r = \sqrt[3]{\frac{24416640 \cdot 3}{4\pi}} = 179.7 \text{ dm}$$

$$\text{Aesfera} = 4\pi r^2 = 4\pi \cdot 179.7^2 = 407012.79 \text{ dm}^2 \approx 407013 \text{ m}^2$$

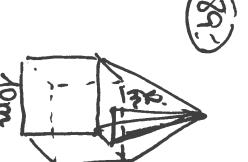
$$92) D = \sqrt{10^2 + 100^2 + 250^2} = 287.23 < 288 \Rightarrow \text{No podemos}$$

$$93) A = P_b \cdot h + A_B = 20 \cdot 3 + 6 \cdot 4 = 84 \text{ m}^2$$

(el solo no se pinta)

- a) Necesitaremos 3 botes
b) $84 : 4 = 21 \text{ m}^2$ cada bote

$$94) a^2 = b^2 + c^2 \\ 179.7^2 = h^2 + \left(\frac{215.25}{2}\right)^2 \Rightarrow h = 143.49 \text{ m}$$



$$95) a^2 = b^2 + c^2 \\ a^2 = 12^2 + 5^2 \\ a = 13 \text{ m}$$

$$A = 5l^2 + \frac{P_b \cdot a}{2} = 5 \cdot 10^2 + \frac{40 \cdot 13}{2} = 760 \text{ m}^2$$



$$96) a^2 = b^2 + c^2 \\ 6^2 = 5^2 + 1^2 \\ g = 5.22 \text{ cm}$$

$$e) \quad x^2 = 4^2 + 4^2 \\ x = 4\sqrt{2}$$

$$A_{\Delta} = 8^2 - \frac{4 \cdot 4}{2} = 56 \text{ cm}^2$$

$$97) V = \text{Vcilindro} - \text{Vesfera} = \pi r^2 h - \frac{4}{3}\pi r^3 = \pi r^2 \cdot 2r - \frac{4}{3}\pi r^3 = 2\pi r^3 - \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$$

$$98) a) V = 3^3 - \frac{\pi \cdot 15^2 \cdot 4}{3} = 12.86 \text{ cm}^3$$

$$b) A_{\Delta} = \frac{4\sqrt{2} \cdot 4\sqrt{2}}{2} = 12.86 \text{ cm}^2$$

$$A_{\Delta} = 3 \cdot 5.6 + 3 \cdot 8^2 + 13 \cdot 8 = 373.86 \text{ cm}^2$$

$$h^2 = 32 - 8 = 24$$

$$h = \sqrt{24} = 4.9 \text{ cm}$$

$$99) 2\pi rH = 2\pi Rh \Rightarrow 6H = 8h \Rightarrow 6(h+3) = 8h \Rightarrow 6h + 18 = 8h \Rightarrow h = 9 \text{ m} \Rightarrow H = 12 \text{ m}$$

$$A_1^{\text{Total}} = 2\pi rH + 2\pi r^2 = 2\pi \cdot 6 \cdot 12 + 2\pi \cdot 6^2 = 216\pi = 648.58 \text{ m}^2$$

$$A_2^{\text{Total}} = 2\pi Rh + 2\pi R^2 = 2\pi \cdot 8 \cdot 9 + 2\pi \cdot 8^2 = 272\pi = 854.51 \text{ m}^2$$

$$A_{\text{Lat}} = 2\pi rh = 2\pi \cdot 6 \cdot 12 = 144\pi = 452.39 \text{ m}^2$$

$$A_{\text{Lat}} = 2\pi Rh = 2\pi \cdot 8 \cdot 9 = 144\pi = 452.39 \text{ m}^2$$

$$100) a) (5 \cdot 6 + 3 \cdot 8) \cdot 3^2 = 486 \text{ cm}^2$$

$$b) \quad \begin{array}{c} 2 \\ \diagdown \\ 3 \\ \diagup \\ a \\ \hline 6 \end{array} \quad \begin{array}{l} a^2 = b^2 + c^2 \\ a^2 = 2^2 + 3^2 \\ a = 3.6 \text{ cm} \end{array}$$

$$= 5 \cdot l^2 + \frac{P_b \cdot a}{2} = 5 \cdot 6^2 + \frac{24 \cdot 3.6}{2} = 223.27 \text{ cm}^2$$

$$c) A = 2\pi r^2 + 2\pi rh + \pi r^2 = \pi r \cdot 6^2 + 2\pi \cdot 6 \cdot 6 + \pi \cdot 6^2 = 192\pi = 603.2 \text{ cm}^2$$

$$d) A = \frac{1}{2}(A_{\text{Cilindro}} + A_{\text{cono}}) + A_{\text{Círculo}} + A_{\text{Rectángulo}} =$$

$$= \frac{1}{2}(2\pi rh + \pi r^2) + \pi r^2 + b \cdot a = \frac{1}{2}(2\pi \cdot 15.5 + \pi \cdot 15.5 \cdot 15.5) + \pi \cdot 15^2 + 15 \cdot 5 = 5043 \text{ cm}^2$$

$$e) \quad \begin{array}{c} x \\ \diagdown \\ 4 \\ \diagup \\ x \\ \hline 4 \end{array} \quad x^2 = 4^2 + 4^2 \\ x = 4\sqrt{2}$$

$$A_{\Delta} = \frac{4\sqrt{2} \cdot 4\sqrt{2}}{2} = 12.86 \text{ cm}^2$$

$$f) \quad \begin{array}{c} 3^3 - \pi \cdot 15^2 \cdot 4 \\ \hline 3 \end{array} = 579 \text{ cm}^3$$

$$g) V = 3^3 - \frac{1}{3}\pi \cdot 15^2 \cdot 3 = 1993 \text{ cm}^3$$

$$\frac{1993}{27} \cdot 100 = 73.82\%$$

