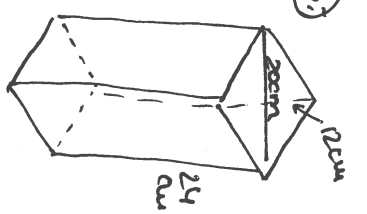


53:



$$a^2 = b^2 + c^2$$

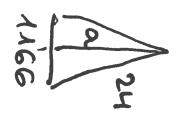
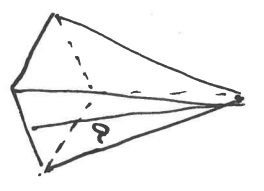
$$1^2 = 6^2 + 10^2$$

$$l = 1166 \text{ cm}$$

$$A = A_L + 2 A_B = P_b \cdot h + 2 \cdot \frac{D \cdot d}{2} =$$

$$= 4 \cdot 1166 \cdot 24 + 20 \cdot 12 = \boxed{1359'54 \text{ cm}^2}$$

b)

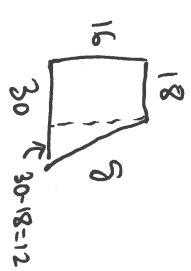
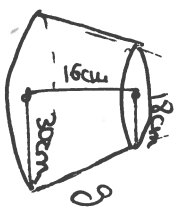


$$a = \sqrt{24^2 - 5'83^2} = 23'28 \text{ cm}$$

$$A = A_L + A_B = \frac{P_b \cdot a}{2} + \frac{D \cdot d}{2} =$$

$$= \frac{4 \cdot 1166 \cdot 23'28}{2} + \frac{20 \cdot 12}{2} = \boxed{662'9 \text{ cm}^2}$$

56:



$$a^2 = b^2 + c^2$$

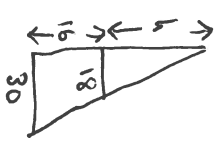
$$9^2 = 16^2 + 12^2$$

$$9 = 20 \text{ cm}$$

$$A = A_L + A_B + A_b = \pi(R+r)g + \pi R^2 + \pi r^2 =$$

$$= \pi(30+18) \cdot 20 + \pi \cdot 30^2 + \pi \cdot 18^2 = 2184\pi =$$

$$\boxed{6861'24 \text{ cm}^2}$$



$V = V_{\text{cono grande}} - V_{\text{cono pequeño}}$

$$\frac{h}{h+16} = \frac{18}{30} \Rightarrow 30h = 18h + 288 \Rightarrow h = 24 \text{ cm}$$

$$\leftarrow H = 40 \text{ cm}$$

$$V = V_{\text{cono grande}} - V_{\text{cono pequeño}} =$$

$$= \frac{1}{3} \pi R^2 H - \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (30 \cdot 40 - 18^2 \cdot 24) = 9408\pi =$$

$$\boxed{29556'1 \text{ cm}^3 = 29556 \ell}$$

81:  $A_{\text{arbo}} = 6a^2 = 216 \Rightarrow a^2 = 36 \Rightarrow a = 6 \text{ cm} \Rightarrow V = a^3 = 6^3 = 216 \text{ cm}^3$

Arbo:  $4\pi r^2 = 216 \Rightarrow r = \sqrt{\frac{216}{4\pi}} = 4'15 \text{ cm}$

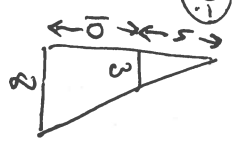
Verfara:  $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi \cdot 4'15^3 = 2985 \text{ cm}^3$

Tiene más volumen la esfera.

3<sup>o</sup> 550

85:  $A_L = \frac{P_b \cdot a}{2} = 80 \Rightarrow \frac{32a}{2} = 80 \Rightarrow a = \frac{80 \cdot 2}{32} = 5 \text{ cm}$

60:

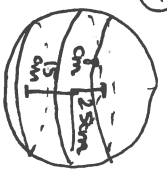


$$\frac{h}{h+10} = \frac{3}{8} \Rightarrow 8h = 3h + 30 \Rightarrow h = 6 \text{ cm} \Rightarrow H = 16 \text{ cm}$$

$$V_{\text{trunco Piramide grande}} = V_{\text{piramide grande}} - V_{\text{piramide pequeña}} =$$

$$= \frac{1}{3} A_B \cdot H - \frac{1}{3} A_b \cdot h = \frac{1}{3} (16^2 \cdot 16 - 6^2 \cdot 6) = \boxed{1293'33 \text{ cm}^3}$$

76:



Arco esférica =  $2\pi r h = 2\pi \cdot 25 \cdot 23 = 1150\pi =$

$$\boxed{3612'83 \text{ cm}^2}$$

81: a)  $V = V_{\text{arbolado}} + V_{\text{piramide}} = abc + \frac{A_b \cdot h}{3} = 4 \cdot 2 \cdot 2 + \frac{2^2 \cdot 2}{3} =$

$$= \frac{3}{186'7 \text{ cm}^3}$$

b)  $V = V_{\text{cilindro}} + V_{\text{cono}} = \pi r^2 h + \frac{\pi r^2 h}{3} =$

$$= \pi \cdot 3^2 \cdot 4 + \frac{\pi \cdot 3^2 \cdot 4}{3} = 48\pi = \boxed{150'8 \text{ cm}^3}$$

c)  $V = V_{\text{cilindro}} - V_{\text{cono}} = \pi r^2 h - \frac{\pi r^2 h}{3} = \pi \cdot 4^2 \cdot 8 - \frac{\pi \cdot 4^2 \cdot 4}{3} = \boxed{335'1 \text{ cm}^3}$

d)  $V = V_{\text{arbo grande}} - 8 \cdot V_{\text{arbo pequeño}} = 9^3 - 8 \cdot 3^3 = \boxed{513 \text{ cm}^3}$

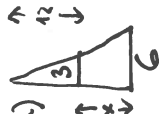
e)  $V = \frac{V_{\text{cilindro}} + V_{\text{cono}}}{2} = \frac{1}{2} (\pi \cdot 15^2 \cdot 5 + \frac{\pi \cdot 15^2 \cdot 5}{3}) = 75\pi = \boxed{2356 \text{ cm}^3}$


f)  $V = V_{\text{arbo}} - V_{\text{piramide}} = 6^3 - \frac{6^2 \cdot 4}{3} = \boxed{168 \text{ cm}^3}$

g)  $V = V_{\text{cubo}} - V_{\text{pirâmide}} = 8^3 - \frac{1}{3} \cdot \frac{4 \cdot 4}{2} \cdot 4 = \boxed{501,33 \text{ cm}^3}$

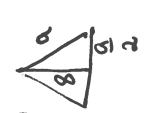
(Pirâmide triangular de base em triângulo retângulo de catetos de 4cm)

h)  $V = V_{\text{esfera}} + V_{\text{cilindro}} = \frac{4}{3} \pi r^3 + \pi r^2 h = \frac{4}{3} \pi \cdot 6^3 + \pi \cdot 6^2 \cdot 7 = 560\pi = \boxed{1696,46 \text{ cm}^3}$

83: a)   $\frac{12x}{12+x} = \frac{3}{6} \Rightarrow 72 = 36 + 3x \Rightarrow x = 12 \Rightarrow H = 24 \text{ cm}$

b)   $h^2 + 2^2 = 6^2$   
 $h = 5,66 \text{ cm}$   
 $H = 11,32 \text{ cm}$   
 $V = \frac{1}{3} \pi (R^2 H - r^2 h) = \frac{1}{3} \pi (4^2 \cdot 11,32 - 2^2 \cdot 5,66) = \boxed{165,96 \text{ cm}^3}$


c)  $5 = \sqrt{a^2 + a^2} \Rightarrow 5 = \sqrt{2}a \Rightarrow a = \frac{5}{\sqrt{2}} = 2,9 \text{ cm}$   
 $V = a^3 = 2a^3 = \boxed{24,06 \text{ cm}^3}$

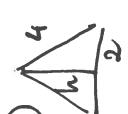
d)   $a^2 = b^2 + c^2$   
 $a^2 = 8^2 + (\frac{a}{2})^2$   
 $a^2 = 64 + \frac{a^2}{4} \Rightarrow \frac{3a^2}{4} = 64 \Rightarrow a^2 = \frac{64 \cdot 4}{3} \Rightarrow a = \frac{16}{\sqrt{3}} = 9,24 \text{ cm}$   
 $h = \sqrt{8^2 - (\frac{8}{3})^2} = 7,54 \text{ cm}$   
 $A = \frac{1}{3} A_B \cdot h = \frac{1}{3} \cdot \frac{9,24 \cdot 8}{2} \cdot 7,54 = 92,87 \text{ cm}^2$


99: Nuevo envase:  $A_B = 66 \text{ cm}^2 \rightarrow A_B = 0,9 \cdot 66 = 59,4 \text{ cm}^2$   
 $h = 15 \rightarrow h = 11 \cdot 15 = 16,5 \text{ cm}$

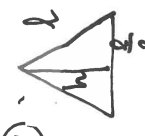
a)  $V_{\text{antigo}} = 11 \cdot 6 \cdot 15 = 990 \text{ cm}^3$   $V_{\text{novo}} = A_B \cdot h = 59,4 \cdot 16,5 = 980,1 \text{ cm}^3$   
É menor!

b) No, Pq pagaria lo mismo por menos  
 c) Ahora:  $(99000 : 0,98) \cdot 1,40 \text{ €} = 141.428,57 \text{ €}$   
 Antes:  $(99000 : 0,99) \cdot 1,40 \text{ €} = 140.000 \text{ €}$

93:   $h = \sqrt{l^2 - 1^2}$   
 $h = \sqrt{3}$   
 $A_{\text{cara}} = \frac{b \cdot h}{2} = \frac{2 \cdot \sqrt{3}}{2} = \sqrt{3}$   
 $A_{\text{tetraedro}} = 4 \cdot A_{\text{cara}} = 4\sqrt{3}$


94:   $h = \sqrt{2^2 - 2^2}$   
 $h = \sqrt{2} = 2\sqrt{3}$   
 $A_{\text{cara}} = \frac{b \cdot h}{2} = \frac{4 \cdot 2\sqrt{3}}{2} = 4\sqrt{3}$   
 $A_{\text{octaedro}} = 8 \cdot A_{\text{cara}} = 8 \cdot 4\sqrt{3} = 32\sqrt{3}$

95:   $h = \sqrt{3^2 - 3^2}$   
 $h = \sqrt{27} = 3\sqrt{3}$   
 $A_{\text{cara}} = \frac{b \cdot h}{2} = \frac{6 \cdot 3\sqrt{3}}{2} = 9\sqrt{3}$   
 $A_{\text{icosaedro}} = 20 \cdot A_{\text{cara}} = 20 \cdot 9\sqrt{3} = 180\sqrt{3}$

96:   $l^2 = h^2 + (\frac{l}{2})^2 \Rightarrow l^2 = h^2 + \frac{l^2}{4} \Rightarrow l^2 - \frac{l^2}{4} = h^2 \Rightarrow h^2 = \frac{3l^2}{4} \Rightarrow h = \frac{l \cdot \sqrt{3}}{2}$   
 $\Rightarrow h = \frac{l \cdot \sqrt{3}}{2} \Rightarrow A_{\text{triângulo equilátero}} = \frac{b \cdot h}{2} = \frac{l \cdot \frac{l \cdot \sqrt{3}}{2}}{2} = \frac{l^2 \sqrt{3}}{4}$   
 a)  $A_{\Delta} = \frac{16\sqrt{3}}{4} = 4\sqrt{3}$   
 $\frac{l^2 \sqrt{3}}{4} = 4\sqrt{3} \Rightarrow l^2 = 16 \Rightarrow l = 4 \text{ cm}$

b)  $A_{\Delta} = \sqrt{3} = \frac{l^2 \sqrt{3}}{4} \Rightarrow l^2 = 4 \Rightarrow l = 2 \text{ cm}$

c)  $A_{\Delta} = \frac{18\sqrt{3}}{8} = \frac{9\sqrt{3}}{4} \Rightarrow \frac{l^2 \sqrt{3}}{4} = \frac{9\sqrt{3}}{4} \Rightarrow l^2 = 9 \Rightarrow l = 3 \text{ cm}$

97:   $\sqrt{2}x = \sqrt{x^2 + (4x)^2 + (2x)^2} = \sqrt{21}x \Rightarrow x = 1 \text{ cm}$   
 $A = P_B \cdot h + 2 \cdot A_B = 12 \cdot 1 + 2 \cdot 4 \cdot 2 = \boxed{28 \text{ cm}^2}$

90:  $V_{\text{cubo}} = a^3 = 125 \Rightarrow a = 5 \text{ cm} \Rightarrow A_{\text{cubo}} = 6a^2 = 6 \cdot 5^2 = 150 \text{ cm}^2$   
 $V_{\text{esfera}} = \frac{4}{3} \pi r^3 = 125 \Rightarrow r = \sqrt[3]{\frac{125 \cdot 3}{4\pi}} = 3,1 \text{ cm}$   
 $A_{\text{esfera}} = 4\pi r^2 = 4\pi \cdot 3,1^2 = 120,9 \text{ cm}^2$

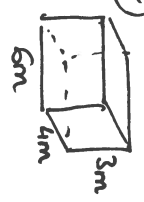
Se necessita menos material con una esfera

91)  $V_{esfera} = \frac{4}{3} \pi r^3 = 24416640 \text{ dm}^3 \Rightarrow r = \sqrt[3]{\frac{24416640 \cdot 3}{4\pi}} = 179,97 \text{ dm}$

$A_{esfera} = 4\pi r^2 = 4\pi \cdot 179,97^2 = 407012,791 \text{ dm}^2 \approx 407013 \text{ m}^2$

96)  $D = \sqrt{100^2 + 100^2 + 250^2} = 287,23 < 288 \Rightarrow$  Nu podemos

81)  $A = P_b \cdot h + A_b = 20 \cdot 3 + 6 \cdot 4 = 84 \text{ m}^2$



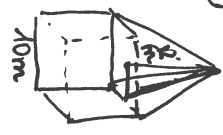
(el suelo no se pinta) a) Necesitamos 3 botes

b) 84:4 = 21 m<sup>2</sup> cada bote

88)  $a^2 = b^2 + c^2$

$179,37^2 = h^2 + \left(\frac{215,25}{2}\right)^2 \Rightarrow h = 143,49 \text{ m}$

89)  $A = 5,8^2 + \frac{P_b \cdot a}{2} = 5 \cdot 10^2 + \frac{40 \cdot 13}{2} = 760 \text{ m}^2$



$a^2 = b^2 + c^2$   
 $a^2 = 12^2 + 5^2$   
 $a = 13 \text{ m}$

101)  $V = V_{cilindro} - V_{esfera} = \pi r^2 h - \frac{4}{3} \pi r^3 = \pi r^2 (2r - \frac{4}{3} r) = \frac{2}{3} \pi r^3$   
 $= 2\pi r^3 - \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3$

110)  $2\pi r H = 2\pi R h \Rightarrow 6H = 8h \Rightarrow 6(h+3) = 8h \Rightarrow 6h + 18 = 8h \Rightarrow h = 9 \text{ m} \Rightarrow H = 12 \text{ m}$

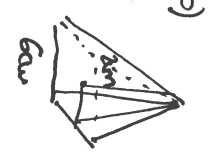
$A_1^{\text{Total}} = 2\pi r H + 2\pi r^2 = 2\pi \cdot 6 \cdot 12 + 2\pi \cdot 6^2 = 216\pi = 678,58 \text{ m}^2$

$A_2^{\text{Total}} = 2\pi R h + 2\pi R^2 = 2\pi \cdot 8 \cdot 9 + 2\pi \cdot 8^2 = 232\pi = 734,51 \text{ m}^2$

$A_{\text{ext}} = 2\pi r H = 2\pi \cdot 6 \cdot 12 = 452,39 \text{ m}^2$

$A_{\text{ext}} = 2\pi R h = 2\pi \cdot 8 \cdot 9 = 452,39 \text{ m}^2$

100) a)  $(5 \cdot 6 + 3 \cdot 8) \cdot 3^2 = 486 \text{ cm}^2$



$a^2 = b^2 + c^2$   
 $a^2 = 2^2 + 3^2$   
 $a = 3,6 \text{ cm}$

$A = 5,8^2 + \frac{P_b \cdot a}{2} = 5 \cdot 6^2 + \frac{24 \cdot 3,6}{2} = 223,29 \text{ cm}^2$

c)  $A = 2\pi r^2 + 2\pi r h + \pi r^2 = 2\pi \cdot 6^2 + 2\pi \cdot 6 \cdot 7 + \pi \cdot 6^2 = 192\pi = 603,2 \text{ cm}^2$

d)  $A = \frac{A}{2} (A_{cilindro} + A_{cono}) + A_{cuadro} + A_{rectangulo} =$

$= \frac{1}{2} (2\pi r h + \pi r g) + \pi r^2 + b \cdot a = \frac{1}{2} (2\pi \cdot 15 \cdot 5 + \pi \cdot 15 \cdot 5 \cdot 2) +$



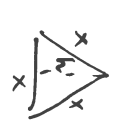
$a^2 = b^2 + c^2$   
 $g^2 = 5^2 + 8^2$   
 $g = 9,76 \text{ cm}$

$+ \pi \cdot 15^2 + 15 \cdot 5 = 504,3 \text{ cm}^2$



$x^2 = 4^2 + 4^2$   
 $x = 4\sqrt{2}$

$A_{cuadro} = 8^2 - \frac{4 \cdot 4}{2} = 56 \text{ cm}^2$



$h + (2 \cdot 2)^2 = (4\sqrt{2})^2$   
 $h^2 = 32 - 8 = 24$   
 $h = \sqrt{24} = 4,9 \text{ cm}$

$A_{\Delta} = \frac{4\sqrt{2} \cdot 4,9}{2} = 13,86 \text{ cm}^2$

$A_T = 3 \cdot 56 + 3 \cdot 8^2 + 13,86 = 373,86 \text{ cm}^2$

98) a)  $V = 3^3 - \frac{\pi \cdot 1,5^3 \cdot 4}{3} = 12,86 \text{ cm}^3$

$\frac{12,86}{27} \cdot 100 = 47,64 \%$

b)  $V = 3^3 - \pi \cdot 1,5^2 \cdot 3 = 5,79 \text{ cm}^3$   
 $\frac{5,79}{27} \cdot 100 = 21,44 \%$

c)  $V = 3^3 - \frac{1}{3} \pi \cdot 1,5^2 \cdot 3 = 19,93 \text{ cm}^3$   
 $\frac{19,93}{27} \cdot 100 = 73,82 \%$

