

$$d) \left(1 + \frac{2}{x}\right) \left(1 + \frac{2}{x}\right) \cdot \frac{x^2 - 4}{2x} = \frac{x-2}{x} \cdot \frac{x+2}{x} \cdot \frac{x^2 - 4}{2x} = \frac{2}{x}$$

$$e) \left(\frac{1}{2} + \frac{x+1}{3x}\right) \frac{12x}{(x-2)^2} = \frac{-3x-2x+2}{6x} \cdot \frac{12x}{(x-2)^2} = \frac{2}{x-2}$$

$$f) \left(\frac{x-3}{x} + \frac{x+3}{3x}\right) \frac{1}{3x-9} = \frac{3(x-3)}{x+3} \cdot \frac{1}{3x-9} = \frac{1}{x+3}$$

$$95) a) \frac{x}{x+1} \cdot \frac{3}{x^2} = \frac{3}{x^2+x}; \quad b) \frac{3x+2}{x-1} \cdot \frac{x+1}{x} = \frac{3x^2+2x}{x^2-1}$$

$$c) \frac{3}{(x-1)^2} : \frac{2}{x-1} = \frac{3}{2x-2}; \quad d) (x+1) \cdot \frac{x^2-1}{2} = \frac{2}{x-1}$$

$$96) a) x - \frac{1}{3}x - \frac{2}{5} \cdot \frac{2}{3}x - 20; \quad b) 0,8x + 0,5y = 60; \quad c) 3(x+1) + 2x = 6$$

$$97) P = \sqrt{x^2+x^2/4} + \sqrt{x^2/4+x^2/4} + \sqrt{x^2+x^2/4} = \frac{x\sqrt{5}}{2} + \frac{x\sqrt{2}}{2} + \frac{x\sqrt{5}}{2} = \frac{x(2\sqrt{5}+\sqrt{2})}{2}$$

$$A = x^2 - \left(\frac{1 \cdot x \cdot x}{2} + \frac{1 \cdot x \cdot x}{2} + \frac{1 \cdot x \cdot x}{2}\right) = x^2 - \frac{x^2}{4} - \frac{x^2}{8} - \frac{x^2}{4} = \frac{3x^2}{4}$$

$$98) f = P_B h + 2A_B = (4x+2)(x+2) + 2x(x+1) = 4x^2 + 8x + 2x + 4 + 2x^2 + 2x = 6x^2 + 12x + 4$$

$$V = x(x+1)(x+2) = x(x^2 + 3x + 2) = x^3 + 3x^2 + 2x$$

$$99) A = A_1 + 2A_B = 2\pi r h + 2\pi r^2 = 2\pi R \cdot 2R + 2\pi R^2 = 4\pi R^2 + 2\pi R^2 = 6\pi R^2$$

$$V = A_B \cdot h = \pi r^2 \cdot h = \pi R^2 \cdot 2R = 2\pi R^3$$

$$100) \text{Diagram: } y = \sqrt{(10-x)^2 + (10-x)^2} = (10-x)\sqrt{2}$$

$$P = 20 + 2x + (10-x)\sqrt{2}$$

$$A = 10 \cdot x + \frac{(x+10)(10-x)}{2} = 10x + \frac{100-x^2}{2} = \frac{-x^2}{2} + 10x + 50$$

$$101) A = xy - (x-4)(y-4) = xy - xy + 4x + 4y - 16 = 4x + 4y - 16$$

$$102) a) \frac{1}{x} + \frac{2}{x^2} = \frac{x+2}{x^2}; \quad b) \frac{3}{x} + \frac{1}{2x} = \frac{5}{2x}; \quad c) \frac{5}{2x} - \frac{3}{x^2} = \frac{5x-6}{2x^2}$$

$$d) \frac{3-x}{x} + \frac{x-1}{x^2} = \frac{3x-x^2+x-1}{x^2} = \frac{-x^2+4x-1}{x^2}; \quad e) 2x + \frac{3}{x-1} = \frac{2x^2+2x+3}{x-1}$$

$$103) a) \frac{1}{6x} + \frac{1}{3x^2} - \frac{1}{2x^3} = \frac{x^2}{6x^3}; \quad f) \frac{2x}{x+1} - x = \frac{-x+x}{x+1}$$

$$b) \frac{2}{x} + \frac{x-1}{x-7} = \frac{2x-14+x^2-x}{x(x-7)} = \frac{x^2+x-14}{x^2-7x}; \quad c) \frac{2}{x} - \frac{3}{x-4} + \frac{x+1}{x-4} = \frac{2x-8-3x+x+1}{x(x-4)} = \frac{x^2-8}{x^2-4x}$$

$$d) \frac{2x}{x-3} - \frac{x-1}{x+3} = \frac{2x^2+6x-(x^2-4x+3)}{(x-3)(x+3)} = \frac{x^2+10x-3}{x^2-9}$$

$$c) \frac{3}{x-1} + \frac{1}{2} + \frac{x}{4} = \frac{12+2x-2+x^2}{4(x-1)} = \frac{x^2+x+10}{4x-4}; \quad f) \frac{3}{x} - \frac{1}{x^2+x} + 2 = \frac{3x+3-1+2x^2+2x}{x^2+x} = \frac{2x^2+5x+2}{x^2+x}$$

$$(104) a) \frac{x+2}{3} \cdot \frac{1}{x+2} = \frac{1}{3}; \quad b) \frac{x-3}{2x} \cdot \frac{x}{x-3} = \frac{x}{2}; \quad c) \frac{3}{x^2-4} \cdot \frac{x+2}{2} = \frac{3}{2x-4}$$

$$d) \frac{(x-1)^2}{x} \cdot \frac{1}{x-1} = \frac{x-1}{x}; \quad e) \frac{5}{x-2} \cdot \frac{x+1}{x-2} = \frac{5}{x-1}; \quad f) \frac{x+5}{5x} \cdot \frac{x+5}{x^2} = \frac{x}{5}$$

$$(105) a) x^3-4x-x(x^2-4)=x(x-2)(x+2); \quad b) 4x^3-4x^2+x=x(4x^2-4x+1)=x(2x-1)^2;$$

$$c) x^4-x^2=x^2(x^2-1)=x^2(x-1)(x+1); \quad d) 3x^4-24x^3+48x^2=3x^2(x^2-8x+16)=3x^2(x-4)^2$$

$$(106) a) 100a+10b+c; \quad b) 100a+10b+c+1; \quad 100a+10b+c-1$$

$$c) 100a+10b+c - (100c+10b+a) = 99a - 99c$$

$$(107) e) \frac{x}{2} + 20 = 3x$$

$$(110) \begin{array}{r} 6 \\ 9 \\ 15 \end{array} \begin{array}{r} x \\ x-3 \\ 5 \cdot 20 \end{array} \begin{array}{r} 6x \\ 9(x-3) \\ 15 \cdot 5 \cdot 20 = 78 \end{array}$$

$$(108) x + 2x + 3 \cdot 2x = 9 \quad (\Rightarrow 9x = 9 \Rightarrow x = 1)$$

$$15 \cdot 5 \cdot 20 = 78$$

$$(109) b) 12x - 8(x+3)$$

$$6x + 9(x-3) = 78 \quad (\Rightarrow x = 7)$$

$$(111) a) (x+a)^2 = (-x-a)^2. \text{ Verdadero: } (5+1)^2 = 6^2 = 36; \quad (-5-1)^2 = (-6)^2 = 36$$

$$b) (x-a)^2 = (a-x)^2. \text{ Verdadero: } (5-1)^2 = 4^2 = 16; \quad (1-5)^2 = (-4)^2 = 16$$

$$c) -(x)^2 = x^2 \text{ falso: } -(5)^2 = -25; \quad 5^2 = 25$$

$$2x \cdot y = 2xy$$

d) falso: si multiplicamos dos monomios nos da otro monomio. \checkmark

e) falso: xyz , x^2yz no son semejantes y tienen las mismas letras

$$f) \text{ falso: } 4xy + (-2xy) = 2xy; \quad 4xy \cdot (-2xy) = -8x^2y^2$$

$$(112) (-2)^3 - 5(-2)^2 - 7(-2) + k = 0 \Rightarrow -8 - 20 + 14 + k = 0 \Rightarrow k = +14$$

$$(113) 1; \quad \frac{x}{x+2} \cdot \frac{x+2}{x} = 1$$

$$(114) a) (a+1)^2 - (a-1)^2 = a^2 + 2a + 1 - (a^2 - 2a + 1) = 4a$$

$$b) 2501^2 - 2499^2 = 4 \cdot 2500 = 10000$$

$$(115) a) (3x+a)(3x-a) + 7 = 9x^2 - a^2 + 7 = 9x^2 - 18 \Rightarrow a^2 = 25 \Rightarrow \boxed{a = \pm 5}$$

$$b) (x-a)^2 + 2xa - 46 = x^2 - 2xa + a^2 + 2xa - 46 = x^2 + a^2 - 46 = x^2 + 18 \Rightarrow a^2 = 64$$

$$(116) a) \sqrt{9x^2} = 3x; \quad b) x(x+1) = x^2 + 1; \quad c) (x-5)^2 = x^2 - 25$$

$$x^2 + x = x^2 + 1 \\ x = 1$$

$$x^2 + 25 - 10x - x^2 = 25 \\ 50 = 10x \rightarrow x = 5$$

$$\boxed{a = \pm 8}$$

117. a) $\left(\frac{3+x}{8} - \frac{5-x}{6} - \frac{x+1}{12}\right) \cdot 24 = 3(3+x) - 4(5-x) - 2(x+1) = 5x - 13$

b) $12\left(\frac{3}{4}(x-1) - \frac{1}{3}(x+1) + \frac{1}{6}\right) = 9(x-1) - 4(x+1) + 2 = 5x - 11$

c) $30\left(\frac{3x-3}{5} - \frac{x+1}{3} + \frac{1}{2}\right) = 6(3x-3) - 10(x+1) + 15 = 8x - 13$

118. a) $x^4 - x^3 - x^2 + x = x(x^3 - x^2 - x + 1) = x(x-1)^2(x+1)$

1	-1	-1	1	
1	1	0	-1	$x^2 - 1 = (x-1)(x+1)$
1	0	-1	0	

b) $3x^3 + 3x^2 - 18x = 3x(x^2 + x - 6) = 3x(x-2)(x+3)$

c) $x^4 - 2x^3 - 13x^2 + 38x - 24 = (x-1)(x-2)(x-3)(x+4)$

1	-2	-13	38	-24
1	1	-1	-14	24
1	-1	-14	24	0
2	2	2	-24	
1	1	-12	0	

$x^2 + x - 12 = (x+4)(x-3)$

d) $x^4 - 3x^3 + 3x^2 - 3x + 2 = (x-1)(x-2)(x+1)^2$

$x^2(x-1)^2(x-3)$

1	-3	3	-3	2
1	1	-2	1	-2
1	-2	1	-2	0
2	2	0	2	
1	0	1	0	

e) $x^5 - 5x^4 + 7x^3 - 3x^2 = x^2(x^3 - 5x^2 + 7x - 3) =$

1	-5	7	-3
1	1	-4	3
1	-4	3	0

$x^2 - 4x + 3 = (x-1)(x-3)$

f) $2x^3 - 2x^2 - 12x - 2x(x^2 - x - 6) - 2x(x-3)(x+2)$

g) $3x^4 + 6x^3 + 6x^2 + 6x + 3 = 3(x+1)^2(x+1)^2$

3	6	6	6	3
-1	-3	-3	-3	-3
3	3	3	3	0
-1	-3	0	-3	
3	0	3	0	

j) $4x^4 - 6x^3 + 2x^2 = 2x^2(2x^2 - 3x + 1) =$
 $= 2x^2 \cdot 2(x-1)(x-\frac{1}{2}) =$
 $= 2x^2(x-1)(2x-1)$

h) $x^4 + x^3 - 7x^2 - x + 6 = (x-1)(x+1)(x-2)(x+3)$

1	1	-7	-1	6
1	1	2	-5	-6
1	2	-5	-6	0
-1	-1	-1	6	
1	1	-6	0	

$x^2 + x - 6 = (x-2)(x+3)$

i) $x^4 + 3x^3 + 4x^2 + 6x + 4 = (x+1)(x+2)(x+2)^2$

1	3	4	6	4
-1	-1	-2	-2	-4
1	2	2	4	0
-2	-2	0	-4	
1	0	2	0	

119) a) $x^3 + 3x^2 - x - 3 = (x-1)(x+1)(x+3)$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -1 & -3 \\ & & 1 & 4 & 3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$$x^2 + 4x + 3 = (x+1)(x+3)$$

b) $x^3 + 3x^2 - 9x - 27 = (x-3)(x+3)^2$

$$\begin{array}{r|rrrr} 3 & 1 & 3 & -9 & -27 \\ & & 3 & 18 & 27 \\ \hline & 1 & 6 & 9 & 0 \end{array}$$

$$x^2 + 6x + 9 = (x+3)^2$$

c) $x^4 + 4x^3 - 6x^2 - 36x - 27 = (x+1)(x-3)(x+3)^2$

$$\begin{array}{r|rrrrr} -1 & 1 & 4 & -6 & -36 & -27 \\ & & -1 & -3 & 9 & 27 \\ \hline & 1 & 3 & -9 & -27 & 0 \\ 3 & & 3 & 18 & 27 & \\ \hline & 1 & 6 & 9 & 0 & \end{array}$$

d) $x^3 + 3x^2 - 4 = (x-1)(x+2)^2$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$$x^2 + 4x + 4 = (x+2)^2$$

e) $x^4 - 3x^3 - 3x^2 + 11x - 6 = (x+2)(x-3)(x-1)^2$

$$\begin{array}{r|rrrrr} -2 & 1 & -3 & -3 & 11 & -6 \\ & & -2 & -10 & -14 & 6 \\ \hline & 1 & -5 & 7 & -3 & 0 \\ 3 & & 3 & -6 & 3 & \\ \hline & 1 & -2 & 1 & 0 & \end{array}$$

$$x^2 - 2x + 1 = (x-1)^2$$

f) $x^3 - 3x^2 = x^2(x-3)$

g) $x^3 - 7x^2 + 15x - 9 = (x-1)(x-3)^2$

$$\begin{array}{r|rrrr} 1 & 1 & -7 & 15 & -9 \\ & & 1 & -6 & 9 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$$x^2 - 6x + 9 = (x-3)^2$$

h) $x^3 - 13x + 12 = (x-1)(x-3)(x+4)$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -13 & 12 \\ & & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$$x^2 + x - 12 = (x+4)(x-3)$$

i) $x^3 - 2x^2 - 15x + 36 = (x-3)^2(x+4)$

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -15 & 36 \\ & & 3 & 3 & -36 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$$x^2 + x - 12 = (x+4)(x-3)$$

j) $x^4 + 4x^3 - 2x^2 - 12x + 9 = (x-1)(x+3)^2$

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & -2 & -12 & 9 \\ & & 1 & 5 & 3 & -9 \\ \hline & 1 & 5 & 3 & -9 & 0 \\ 1 & & 1 & 6 & 9 & \\ \hline & 1 & 6 & 9 & 0 & \end{array}$$

$$x^2 + 6x + 9 = (x+3)^2$$

$$k) x^3 + 7x^2 + 16x + 12 = (x+2)^2(x+3)$$

$$\begin{array}{r|rrrr} 1 & 7 & 16 & 12 & \\ -2 & & -2 & -10 & -12 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$$x^2 + 5x + 6 = (x+2)(x+3)$$

$$l) x^3 + 4x^2 + x - 6 = (x-1)(x+2)(x+3)$$

$$\begin{array}{r|rrrr} 1 & 4 & 1 & -6 & \\ 1 & & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$$m) x^3 - 2x^2 - 5x + 6 = (x-1)(x+2)(x-3)$$

$$\begin{array}{r|rrrr} 1 & -2 & -5 & 6 & \\ 1 & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$$x^2 - x - 6 = (x-3)(x+2)$$

$$n) x^3 - x^2 - 2x = x(x^2 - x - 2) = x(x+1)(x-2)$$

$$\bar{n}) x^4 - 5x^3 + 3x^2 + 9x = x(x^3 - 5x^2 + 3x + 9) = x(x+1)(x-3)^2$$

$$\begin{array}{r|rrrr} 1 & -5 & 3 & 9 & \\ -1 & & -1 & 6 & -9 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$$x^2 - 6x + 9 = (x-3)^2$$

$$o) x^3 + 5x^2 + 7x + 3 = (x+1)^2(x+3)$$

$$\begin{array}{r|rrrr} 1 & 5 & 7 & 3 & \\ 1 & & -1 & -4 & -3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$$x^2 + 4x + 3 = (x+1)(x+3)$$