Solucións dos exercicios das páxinas 329 a 331:

1 **Halla:**

a) $\int x^4\,dx$

b) $\int (5x^3 - 8x^2 + 2x - 3)\,dx$

c) $\int \sqrt[3]{x}\,dx$

d) $\int \frac{1}{\sqrt{x}}\,dx$

e) $\int \frac{1}{\sqrt[5]{x^2}}$

f) $\int \frac{3}{x^2}$

g) $\int \frac{5}{6x^4}\,dx$

h) $\int \frac{\sqrt[3]{2x}}{\sqrt{3x}}\,dx$

i) $\int \frac{\sqrt[3]{x} + \sqrt{5x^3}}{3x}\,dx$

j) $\int (\sqrt{5}x - 3)^4\,dx$

k) $\int \sqrt[3]{(7x-6)^2}\,dx$

l) $\int \frac{5x^3 + 6x^2 - \sqrt{2}x + \sqrt{3}}{x}\,dx$

m) $\int \frac{2x^4 - 6x^3 + 5x}{x+2}\,dx$

n) $\int \frac{5dx}{6-4x}$

ñ) $\int \frac{2x^4 + 6x - 3}{x-2}\,dx$

o) $\int \frac{7x^4 - 5x^2 + 3x - 4}{x^2}\,dx$

a) $\int x^4\,dx = \frac{x^5}{5} + k$

b) $\int (5x^3 - 8x^2 + 2x - 3)\,dx = 5\int x^3 dx - 8\int x^2\,dx + 2\int x\,dx - 3\int dx = \frac{5x^4}{4} - \frac{8x^3}{3} + x^2 - 3x + k$

c) $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \frac{x^{(1/3)+1}}{\frac{1}{3}+1} + k = \frac{3}{4}x^{4/3} = \frac{3x\sqrt[3]{x}}{4} + k$

d) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{-(1/2)+1}}{-\frac{1}{2}+1} + k = 2x^{1/2} + k = 2\sqrt{x} + k$

e) $\int \frac{1}{\sqrt[5]{x^2}}\,dx = \int x^{-2/5}\,dx = \frac{x^{-(2/5)+1}}{-\frac{2}{5}+1} + k = \frac{5}{3}x^{3/5} + k = \frac{5\sqrt[5]{x^3}}{3} + k$

f) $\int \frac{3}{x^2}\,dx = 3\int x^{-2}\,dx = 3\frac{x^{-2+1}}{-2+1} + k = -\frac{3}{x} + k$

g) $\int \frac{5}{6x^4}\,dx = \frac{5}{6}\int x^{-4}\,dx = \frac{5}{6} \cdot \frac{x^{-4+1}}{-4+1} + k = -\frac{5}{18x^3} + k$

h) $\int \frac{\sqrt[3]{2x}}{\sqrt{3x}}\,dx = \frac{\sqrt[3]{2}}{\sqrt{3}}\int \frac{x^{1/3}}{x^{1/2}}\,dx = \frac{\sqrt[3]{2}}{\sqrt{3}}\int x^{-1/6}\,dx = \frac{\sqrt[3]{2}}{\sqrt{3}} \cdot \frac{x^{-(1/6)+1}}{-\frac{1}{6}+1} + k = \frac{6\sqrt[3]{2}}{5\sqrt{3}}\sqrt[6]{x^5} + k$

i) $\int \frac{\sqrt[3]{x} + \sqrt{5x^3}}{3x}\,dx = \int \frac{x^{1/3}}{3x}\,dx + \int \frac{\sqrt{5}x^{3/2}}{3x}\,dx = \frac{1}{3}\int x^{-2/3}\,dx + \frac{\sqrt{5}}{3}\int x^{1/2}\,dx =$

$= \frac{1}{3} \cdot \frac{x^{1/3}}{\frac{1}{3}} + \frac{\sqrt{5}}{3} \cdot \frac{x^{3/2}}{\frac{3}{2}} + k = \sqrt[3]{x} + \frac{2\sqrt{5x^3}}{9} + k$

j) $\int (\sqrt{5}x - 3)^4\, dx = \frac{1}{\sqrt{5}} \cdot \frac{(\sqrt{5}\,x - 3)^{4+1}}{4+1} + k = \frac{(\sqrt{5}\,x - 3)^5}{5\sqrt{5}} + k$

k) $\int \sqrt[3]{(7x-6)^2}\, dx = \int (7x-6)^{2/3}\, dx = \frac{1}{7} \cdot \frac{(7x-6)^{(2/3)+1}}{\frac{2}{3}+1} + k = \frac{3}{35}(7x-6)^{5/3} + k = \frac{3\,(7x-6)\sqrt[3]{(7x}}{35}$

l) $\int \frac{5x^3 + 6x^2 - \sqrt{2}x + \sqrt{3}}{x}\, dx = \int \left(5x^2 + 6x - \sqrt{2} + \frac{\sqrt{3}}{3}\right) dx = \frac{5x^3}{3} + 3x^2 - \sqrt{2}x + \sqrt{3}\, ln\,|x| + k$

m) $\int \frac{2x^4 - 6x^3 + 5x}{x+2}\, dx = \int \left(2x^3 - 10x^2 + 20x - 35 + \frac{70}{x+2}\right) dx = \frac{x^4}{2} - \frac{10x^3}{3} + 10x^2 - 35x + 70\, ln\,|x +$

n) $\int \frac{5}{6-4x}\, dx = -\frac{5}{4}\, ln\,|6 - 4x| + k$

ñ) $\int \frac{2x^4 + 6x - 3}{x-2}\, dx = \int \left(2x^3 + 4x^2 + 8x + 22 + \frac{41}{x-2}\right) dx = \frac{x^4}{2} + \frac{4x^3}{3} + 4x^2 + 22x + 41\, ln\,|x-2| + k$

o) $\int \frac{7x^4 - 5x^2 + 3x - 4}{x^2}\, dx = \int \left(\frac{7x^4}{x^2}\right) dx - \int \left(\frac{5x^2}{x^2}\right) dx + \int \left(\frac{3x}{x^2}\right) dx - \int \left(\frac{4}{x^2}\right) dx =$

$= \int 7x^2\, dx - \int 5\, dx + \int \frac{3}{x}\, dx - \int \frac{4}{x^2}\, dx = \frac{7x^3}{3} - 5x + 3\, ln\,|x| + \frac{4}{x} + k$

2 **a)** $\int (3x - 5\, tg\, x)\, dx$ **b)** $\int (5\cos x + 3^x)\, dx$ **c)** $\int (3\, tg\, x - 5\cos x)\, dx$ **d)** $\int (10^x - 5^x)\, dx$

a) $\int (3x - 5\, tg\, x)\, dx = 3\int x\, dx - 5\int tg\, x\, dx = \frac{3x^2}{2} - 5(-ln\,|cos\, x|) + k = \frac{3x^2}{2} + 5\, ln\,|cos\, x| + k$

b) $\int (5\cos x + 3^x)\, dx = 5\int cos\, x\, dx + \int 3^x\, dx = 5 sen\, x + \frac{3^x}{ln\, 3} + k$

c) $\int (3\, tg\, x - 5\cos x)\, dx = 3\int tg\, x\, dx - 5\int cos\, x\, dx = 3(-ln\,|cos\, x|) - 5 sen\, x + k = -3 ln\,|cos\, x| - 5 sen\, x + k$

d) $\int (10^x - 5^x)\, dx = \frac{10^x}{ln\, 10} - \frac{5^x}{ln\, 5} + k$

3 **a)** $\int \frac{3}{x^2+1}\, dx$ **b)** $\int \frac{2x}{x^2+1}\, dx$ **c)** $\int \frac{x^2-1}{x^2+1}\, dx$ **d)** $\int \frac{(x+1)^2}{x^2+1}\, dx$

a) $\int \frac{3}{x^2+1}\, dx = 3\, arctg\, x + k$

b) $\int \frac{2x}{x^2+1}\, dx = ln\,|x^2 + 1| + k$

c) $\int \frac{x^2-1}{x^2+1}\, dx = \int \left(1 + \frac{-2}{x^2+1}\right) dx = x - 2 arctg\, x + k$

d) $\int \frac{(x+1)^2}{x^2+1}\, dx = \int \frac{x^2+2x+1}{x^2+1}\, dx = \int \left(1 + \frac{2x}{x^2+1}\right) dx = x + ln\,|x^2+1| + k$

4 a) $\int sen^2 x\, dx$ b) $\int \frac{dx}{1+9x^2}$ c) $\int \frac{dx}{1+8x^2}$

d) $\int \frac{dx}{25+9x^2}$ e) $\int \frac{dx}{3+2x^2}$ f) $\int \frac{dx}{\sqrt{1-9x^2}}$

g) $\int \frac{dx}{\sqrt{1-8x^2}}$ h) $\int \frac{dx}{\sqrt{25-9x^2}}$ i) $\int \frac{dx}{\sqrt{3-2x^2}}$

j) $\int e^{5x-2}\, dx$

a) Restando las ecuaciones del ejercicio resuelto 2.a) de esta página, obtenemos que $1 - cos\, 2x = 2sen^2\, x$.

$$\int sen^2\, x\, dx = \int \frac{1-cos\, 2x}{2}\, dx = \int \frac{1}{2}\, dx - \frac{1}{2}\int cos\, 2x\, dx = \frac{x}{2} - \frac{1}{2}\, \frac{sen\, 2x}{2} + k = \frac{x}{2} - \frac{sen\, 2x}{4} + k$$

b) $\int \frac{dx}{1+9x^2} = \int \frac{dx}{1+(3x)^2} = \frac{1}{3}\, arc\, tg\, 3x + k$

c) $\int \frac{dx}{1+8x^2} = \int \frac{dx}{1+(\sqrt{8}\, x)^2} = \frac{1}{\sqrt{8}}\, arc\, tg\, \sqrt{8}x + k$

d) $\int \frac{dx}{25+9x^2} = \frac{1}{25}\int \frac{dx}{1+\frac{9}{25}x^2} = \frac{1}{25}\int \frac{dx}{1+\left(\frac{3}{5}x\right)^2} = \frac{1}{25}\cdot \frac{1}{\frac{3}{5}} arc\, tg\, \frac{3x}{5} + k = \frac{1}{15} arc\, tg\, \frac{3x}{5} + k$

e) $\int \frac{dx}{3+2x^2} = \frac{1}{3}\int \frac{dx}{1+\frac{2}{3}x^2} = \frac{1}{3}\int \frac{dx}{1+\left(\sqrt{\frac{2}{3}}x\right)^2} = \frac{1}{3}\cdot \frac{1}{\sqrt{\frac{2}{3}}}\, arc\, tg\, \sqrt{\frac{2}{3}}\, x + k = \frac{\sqrt{6}}{6} arc\, tg\, \sqrt{\frac{2}{3}}\, x + k$

f) $\int \frac{dx}{\sqrt{1-9x^2}} = \int \frac{dx}{\sqrt{1-(3x)^2}} = \frac{1}{3}\, arc\, sen\, 3x + k$

g) $\int \frac{dx}{\sqrt{1-8x^2}} = \int \frac{dx}{\sqrt{1-(\sqrt{8}\, x)^2}} = \frac{1}{\sqrt{8}}\, arc\, sen\, \sqrt{8}\, x + k$

h) $\int \frac{dx}{\sqrt{25-9x^2}} = \frac{1}{5}\int \frac{dx}{\sqrt{1-\frac{9}{25}x^2}} = \frac{1}{5}\int \frac{dx}{\sqrt{1-\left(\frac{3}{5}x\right)^2}} = \frac{1}{5}\cdot \frac{1}{\frac{3}{5}}\, arc\, sen\, \frac{3x}{5} + k = \frac{1}{3}\, arc\, sen\, \frac{3x}{5} + k$

i) $\int \frac{dx}{\sqrt{3-2x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{1-\frac{2}{3}x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{1-\left(\sqrt{\frac{2}{3}}x\right)^2}} = \frac{1}{\sqrt{3}}\cdot \frac{1}{\sqrt{\frac{2}{3}}}\, arc\, sen\, \sqrt{\frac{2}{3}}\, x + k = \frac{1}{\sqrt{2}}\, arc\, sen\, \sqrt{\frac{2}{3}}\, x + k$

j) $\int e^{5x-2}\, dx = \frac{1}{5}\, e^{5x-2} + k$

Vídeos con explicacións para facer os exercicios propostos na 233:

Integrais tipo potencia: **https://www.youtube.com/watch?v=UuWtTOn-4VY**

Integrais tipo logaritmo neperiano: **https://youtu.be/vc2fE9ytbJ8**

Integrales tipo exponencial: **https://youtu.be/ogKgzt6W_d0**

*Integrales tipo seno y coseno:***https://youtu.be/oW5iAuOeNXg**

Integrales tipo arcotangente: **https://youtu.be/sFi1JX2cRrs**