MATHEMATICS 4º ESO 29/11/2010

UNITS 7 SIMILARITY AND TRIGONOMETRY
8 SOLVING TRIANGLES

TOTAL	SUM	MARKS
10		

NAME	GROUP

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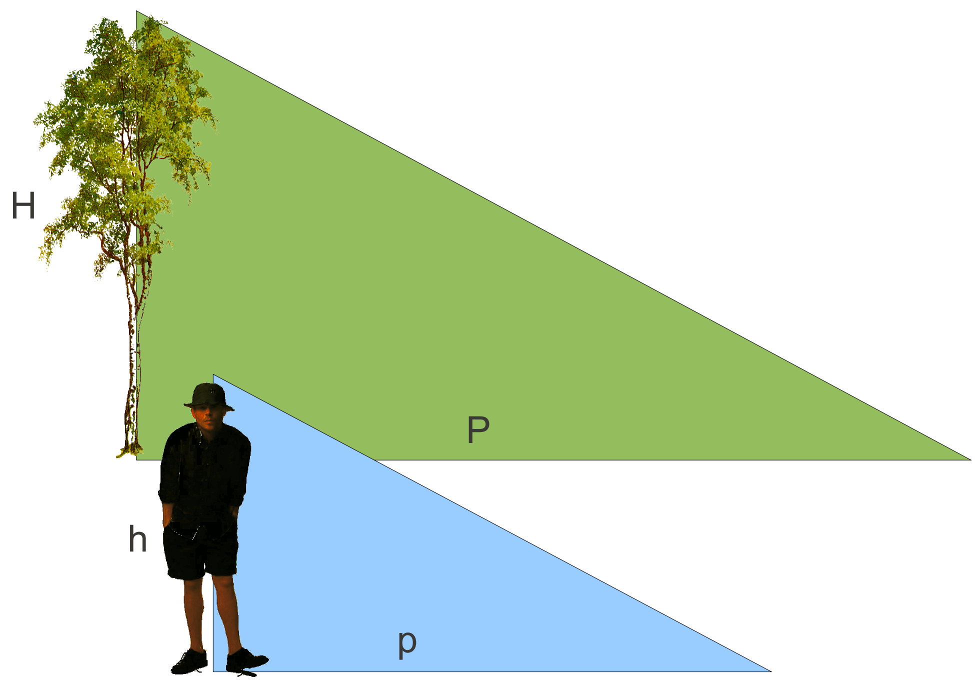
1. The measurement of the projected shadow of a tree equals $20\ m$ at the same time as your own shadow length equals $2\ m$. Find out the height of the tree.
Note: Use your own real height.



Let $P=20\ m$ and $p=2\ m$ be the two projections (shadows) of the tree and my own, respectively, and let $h=1{,}67\ m$ be my own height.

The two triangles are similar triangles because the incident sun rays determine the same angle at the top of both the tree and me, and as they are right angles triangles, the three angles of each are equal (1st criterion of similarity.

Therefore, the homologue sides are proportional and we have:

$$\frac{H}{h}=\frac{P}{p} \Leftrightarrow \frac{H}{1{,}67}=\frac{20}{2} \Leftrightarrow H=\frac{20\cdot 1{,}67}{2}=10\cdot 1{,}67=16{,}7\ m$$

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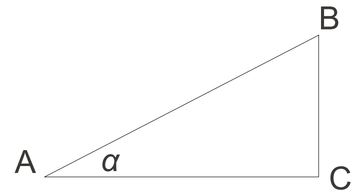
2. Solve the triangle beside, where $\sin\alpha = 0{,}6$ and $\overline{AC} = 8$.



From the Pythagorean identity we can state that:

$$\sin^2\alpha + \cos^2\alpha = 1 \iff \cos^2\alpha = 1 - \sin^2\alpha = 1 - 0{,}6^2 = 1 - 0{,}36 = 0{,}64 \iff$$

$$\iff \cos\alpha = \pm\sqrt{0{,}64} = \pm 0{,}8$$

Since α belongs to the first quadrant, we choose the positive value of the square root, so

$$\cos\alpha = 0{,}8$$

Therefore: $\cos\alpha = \frac{AC}{AB} = \frac{8}{AB} \iff AB = \frac{8}{\cos\alpha} = \frac{8}{0{,}8} = 10$

And using the Pythagorean Theorem we have:

$$AC^2 + BC^2 = AB^2 \iff BC = \sqrt{AB^2 - AC^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$$

Finally, using the calculator, $\alpha = \operatorname{asin} 0{,}6 \approx 37^o$ and the angle on B is $\beta \approx 90^o - 37^o = 53^o$

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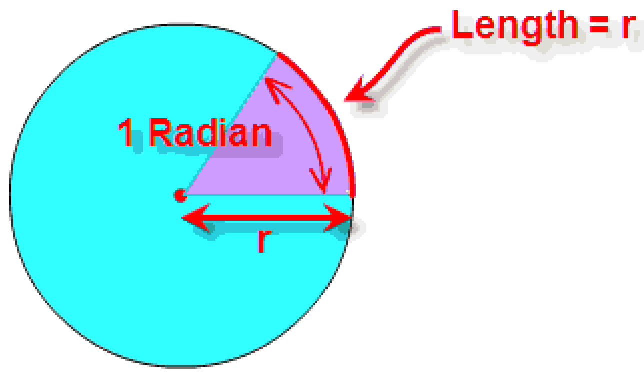
3. i. Explain briefly what a radian is. Sketch a graph with your explanation.
ii. Evaluate the equivalence in radians for the angle of 195^o .
iii. Evaluate the equivalence in degrees for the angle of $3\ rad$.

i. One radian is defined as the angle containing an arc of the same lenght as the radius of the circumference.



ii. As we know that 360^o corresponds to $2\pi\ rad$, we can state the following proportion:

$$\begin{matrix} 360^o & \rightarrow & 2\pi\ \ radian \\ 195^o & \rightarrow & x\ \ radian \end{matrix}\ \text{, and therefore}\ x=\frac{2\pi\cdot 195}{360}=\frac{13\cdot\pi}{12}\approx 3{,}40\ rad\ .$$

iii. $$\begin{matrix} 2\pi\ \ radian & \rightarrow & 360^o \\ 3\ \ radian & \rightarrow & x \end{matrix}\ \text{, and therefore}\ x=\frac{3\cdot 360^o}{2\pi}=\frac{540^o}{\pi}\approx 171^o\ 53'\ 14''\ .$$

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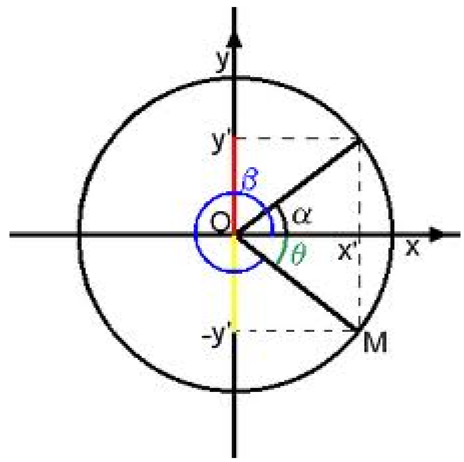
4. i. What's the meaning of 'reducing angles to the first quadrant'? Give some examples.
   ii. Find out which angles belonging to the first quadrant can the following angles be reduced to: 120^o, 255^o e 315^o.
   iii. Evaluate the trigonometric ratios (sine, cosine and tangent) of the angles of 200^o and 290^o, as we know that $\cos 70^o = 0{,}34$.

i. The meaning of 'reducing angles to the first quadrant' is that, given an angle laying on the second, third or fourth quadrants, it's always possible to find out another angle laying on the first quadrant which has the same trigonometric but possibly the sign.

As an example, the angle β is laying on the 4th quadrant and its trigonometric ratios are the same as those of the angle α, in the following sense:



- their sines are opposite: $\sin\beta = -\sin\alpha$
- their cosines are equal: $\cos\beta = \cos\alpha$
- their tangents are also oppposite: $\frac{\sin\beta}{\cos\beta} = \frac{-\sin\alpha}{\sin\alpha} \Rightarrow tg\,\beta = -tg\,\alpha$

ii. 120^o is laying on the 2nd quadrant and it can be reduced to the angle of 60^o;
255^o is laying on the 3rd quadrant and it can be reduced to the angle of 75^o;
315^o is laying on the 4th quadrant and it can be reduced to the angle of 45^o.

iii. Thus 200^o is laying on the 3rd quadrant and it can be reduced to the angle of 20^o and 290^o is laying on the 4th quadrant and it can be reduced to the angle of 70^o, we know that:

$\sin 200^o = -\sin 20^o$ and $\cos 200^o = -\cos 20^o$

$\sin 290^o = -\sin 70^o$ and $\cos 290^o = \cos 70^o$

And we also know that $\sin 20^o = \cos 70^o = 0{,}34$ and $\cos 20^o = \sin 70^o = \sqrt{1-0{,}34^2} = 0{,}94$ because they're complementary angles.

Finally:

$\sin 200^o = -\sin 20^o = -0{,}34$, $\cos 200^o = -\cos 20^o = -0{,}94$ and $tg\,200^o = \frac{-0{,}34}{-0{,}94} = 0{,}36$

$\sin 290^o = -\sin 70^o = -0{,}94$, $\cos 290^o = \cos 70^o = 0{,}34$ and $tg\,290^o = \frac{-0{,}94}{0{,}34} = -2{,}76$

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5. Solve the trigonometric equation $sin^2 x - cos^2 x = -\frac{1}{2}$.

Thus $sin^2 x + cos^2 x = 1 \Leftrightarrow cos^2 x = 1 - sin^2 x$ we have:

$$sin^2 x - cos^2 x = -\frac{1}{2} \Leftrightarrow sin^2 x - (1 - sin^2 x) = -\frac{1}{2} \Leftrightarrow sin^2 x - 1 + sin^2 x = -\frac{1}{2} \Leftrightarrow$$
$$\Leftrightarrow 2 \cdot sin^2 x = 1 - \frac{1}{2} \Leftrightarrow 2 \cdot sin^2 x = \frac{1}{2} \Leftrightarrow sin^2 x = \frac{1}{4} \Leftrightarrow sin x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

Since $sin x = \pm\frac{1}{2}$ we solve for x :

$x = asin\left(\frac{1}{2}\right) \Rightarrow x_1 = 30^o \;\; or \;\; x_2 = 150^o$ or also
$x = asin\left(-\frac{1}{2}\right) \Rightarrow x_3 = 210^o \;\; or \;\; x_4 = 330^o$.

Finally when applying 'rounds' to these four solutions we have:

$$\begin{cases} x_1 = 30^o + k \cdot 360^o \\ x_2 = 150^o + k \cdot 360^o \\ x_3 = 210^o + k \cdot 360^o \\ x_4 = 330^o + k \cdot 360^o \end{cases}$$

It's possible to synthesize those four solutions in two: $\begin{cases} x_1 = 30^o + k \cdot 180^o \\ x_2 = 150^o + k \cdot 180^o \end{cases}$.

Or if preferred, in radians: $\begin{cases} x_1 = \frac{\pi}{6} + k \cdot \pi \\ x_2 = \frac{5 \cdot \pi}{6} + k \cdot \pi \end{cases}$

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6. i. What's the difference between an equation and an identity? Give an example in trigonometric terms.

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ii. Proof the following trigonometric identity: $\frac{1+\sin\alpha}{\cos\alpha}=\frac{\cos\alpha}{1-\sin\alpha}$.

i. Despite both are equalities, a trigonometric equation is true for only some particular values of the unknown(s), and the identity is always true for every value of the unknown(s).

Example:

$\sin^2\alpha+\cos^2\alpha=1$ (the Pythagorean identity): this equality is true for every value of α .

$\sin^2 x-\cos^2 x=-\frac{1}{2}$: this equality is true only for some x-values (as we can see in the preceding exercise).

ii. $\frac{1+\sin\alpha}{\cos\alpha}=\frac{\cos\alpha}{1-\sin\alpha} \Leftrightarrow (1+\sin\alpha)\cdot(1-\sin\alpha)=\cos\alpha\cdot\cos\alpha \Leftrightarrow 1-\sin^2\alpha=\cos^2\alpha \Leftrightarrow$
$\Leftrightarrow 1=\sin^2\alpha+\cos^2\alpha$

As we got the Pythagorean identity, the first equality is proved for every value of α , and therefore it's an identity.

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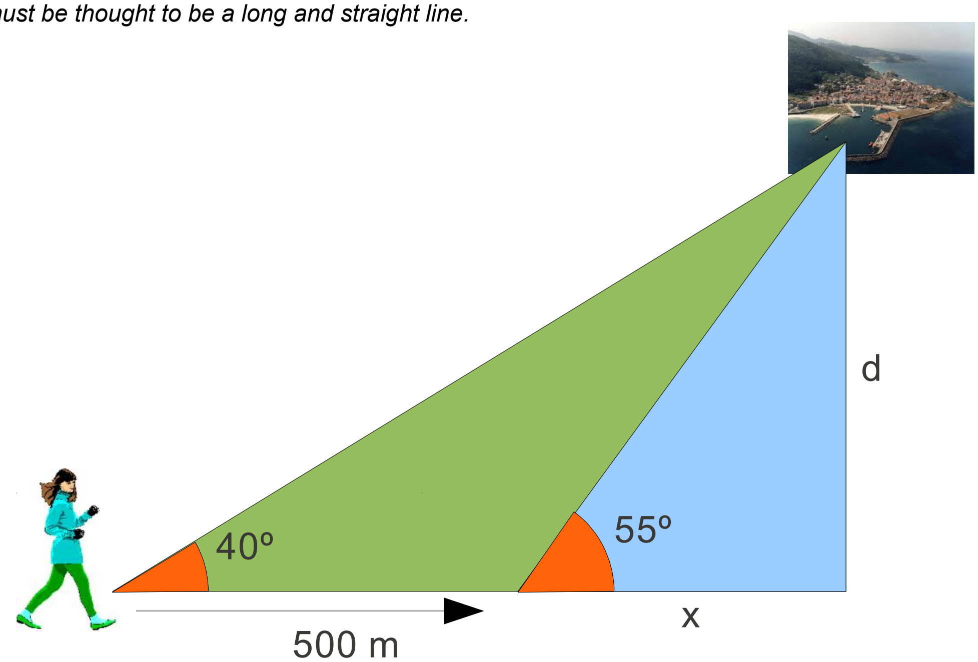
7. As we go walking along the seaside we can see the Porto do Son with an angle of 40^o to the left handside from the direction ahead. When we walk $500\ m$ in the same direction, the angle changes to 55^o. Find out the distance from the coast line to Porto do Son.
Note: The coast must be thought to be a long and straight line.



For the tangents of the two angles, we have the following system of two equations in two unknowns d and h:

$$\begin{cases} tg\,40^o = \dfrac{d}{x+500} \\ tg\,55^o = \dfrac{d}{x} \end{cases} \Leftrightarrow \begin{cases} d = (x+500)\cdot tg\,40^o \\ d = x\cdot tg\,55^o \end{cases}$$

And therefore: $$\begin{cases} tg\,40^o = \dfrac{d}{x+500} \\ tg\,55^o = \dfrac{d}{x} \end{cases} \Leftrightarrow \begin{cases} d = (x+500)\cdot tg\,40^o \\ d = x\cdot tg\,55^o \end{cases}$$

$$\Leftrightarrow\ 500\cdot tg\,40^o = x\cdot tg\,55^o - x\cdot tg\,40^o \ \Leftrightarrow\ 500\cdot tg\,40^o = x\cdot(tg\,55^o - tg\,40^o) \ \Leftrightarrow$$

$$\Leftrightarrow\ x = \frac{500\cdot tg\,40^o}{tg\,55^o - tg\,40^o} \approx \frac{500\cdot 0{,}84}{1{,}43-0{,}84} \approx \frac{419{,}55}{0{,}59} \approx 847{,}40\ m$$

And then the distance is $d = x\cdot tg\,55^o \approx 847{,}40\cdot 1{,}43 \approx 1.210{,}22\ m$

Mínimos 4º ESO

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☐	*Operacións con fraccións e decimais*	FICHA	☐	*Suma, resta, multiplicación e división de números naturais*	FICHA	☐	*Igualdades notábeis*	FICHA
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