$\boxed{1}$ a) $\dfrac{\sqrt[3]{a\sqrt{a}}}{a-\sqrt{a}} - \dfrac{\sqrt{a}}{a-1} = \dfrac{\sqrt[3]{\sqrt{a^2 a}}}{a-\sqrt{a}} - \dfrac{\sqrt{a}}{a-1} =$

$= \dfrac{\sqrt[6]{a^3}}{a-\sqrt{a}} - \dfrac{\sqrt{a}}{a-1} = \dfrac{\sqrt{a}}{a-\sqrt{a}} - \dfrac{\sqrt{a}}{a-1} =$

$= \dfrac{\sqrt{a}(a+\sqrt{a})}{a^2-a} - \dfrac{\sqrt{a}}{a-1} = \dfrac{a\sqrt{a}+a}{a^2-a} - \dfrac{\sqrt{a}}{a-1} =$

$= \dfrac{\cancel{a}(\sqrt{a}+1)}{\cancel{a}(a-1)} - \dfrac{\sqrt{a}}{a-1} = \dfrac{\sqrt{a}+1-\sqrt{a}}{a-1} = \dfrac{1}{a-1}$

b) $(1+\sqrt{5})^4 - (2-\sqrt{5})^2 = (1+\sqrt{5})^2 \cdot (1+\sqrt{5})^2 - (2-\sqrt{5})^2 =$

$= (6+2\sqrt{5})^2 - (9-4\sqrt{5}) = 36+20-24\sqrt{5}-9+4\sqrt{5} =$

$= 47-20\sqrt{5}$

c) $\sqrt{2}\cdot\sqrt[4]{3} - \sqrt[4]{\dfrac{12}{625}} = \sqrt[4]{2^2\cdot 3} - \dfrac{\sqrt[4]{12}}{5} =$

$= \sqrt[4]{12} - \dfrac{1}{5}\sqrt[4]{12} = \dfrac{4}{5}\sqrt[4]{12}$

d) $\dfrac{2}{3}\sqrt{\dfrac{50}{27}} - \dfrac{1}{5}\sqrt{24} = \dfrac{2}{3}\cdot\dfrac{5}{3}\sqrt{\dfrac{2}{3}} - \dfrac{2}{5}\sqrt{6} =$

$= \dfrac{10}{9}\cdot\dfrac{\sqrt{6}}{3} - \dfrac{2}{5}\sqrt{6} = \left(\dfrac{10}{27} - \dfrac{2}{5}\right)\sqrt{6} = \dfrac{-4}{135}\sqrt{6}$

e) $1'\overset{\frown}{7}^{-0'5} \cdot 81^{-0'\overset{\frown}{3}} = \left(\dfrac{16}{9}\right)^{-1/2} \cdot 81^{-1/3} = \left(\dfrac{9}{16}\right)^{1/2} \cdot \left(\dfrac{1}{81}\right)^{1/3}$

$= \dfrac{3}{4}\cdot\dfrac{1}{3}\sqrt[3]{\dfrac{1}{3}} = \dfrac{1}{4}\cdot\dfrac{\sqrt[3]{9}}{3} = \dfrac{\sqrt[3]{9}}{12}$

f) $$\frac{2}{\sqrt{5-\sqrt{3}}} = \frac{2\sqrt{5+\sqrt{3}}}{\sqrt{(5-\sqrt{3})(5+\sqrt{3})}} = \frac{2\sqrt{5+\sqrt{3}}}{\sqrt{22}}$$

$$= \frac{2\sqrt{22}\cdot\sqrt{5+\sqrt{3}}}{22} = \frac{\sqrt{110+22\sqrt{3}}}{11}$$

g) $$\frac{2\sqrt{3}}{\sqrt{6}-\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}-3} = \frac{\cancel{2}\sqrt{3}(\sqrt{6}+\sqrt{2})}{\cancel{4}\,2} -$$

$$- \frac{(\sqrt{3}-1)(2\sqrt{2}+3)}{(-1)} = \frac{3\sqrt{2}+\sqrt{6}}{2} + (\sqrt{3}-1)(2\sqrt{2}+3)$$

$$= \frac{3}{2}\sqrt{2} + \frac{1}{2}\sqrt{6} + 2\sqrt{6} + 3\sqrt{3} - 2\sqrt{2} - 3 =$$

$$= \frac{3}{2}\sqrt{2} + \frac{5}{2}\sqrt{6} + 3\sqrt{3} - 3$$

h) $$\frac{\left(\sqrt{4-\sqrt{15}} - \sqrt{4+\sqrt{15}}\right)^2}{\sqrt{6}-\sqrt{3}} = \frac{4-\sqrt{15}+4+\sqrt{15}-2\sqrt{16-15}}{\sqrt{6}-\sqrt{3}} =$$

$$= \frac{6}{\sqrt{6}-\sqrt{3}} = \frac{6(\sqrt{6}+\sqrt{3})}{3} = 2(\sqrt{6}+\sqrt{3})$$

$\boxed{2}$ a) (I) $(z_1 - \overline{z}_1) \cdot z_4 = 4i \cdot \sqrt{2}i = -4\sqrt{2}$

$z_1 - \overline{z}_1 = 2\,\mathrm{Im}(z_1)\, i$

(II) $$\frac{z_2}{z_3} = \frac{2-2i}{2\sqrt{3}-2i} = \frac{2\sqrt{2}_{-45^\circ}}{4_{-30^\circ}} = \left(\frac{2\sqrt{2}}{4}\right)_{-15^\circ} = \left(\frac{\sqrt{2}}{2}\right)_{-15^\circ}$$

$|z_2| = 2\sqrt{2}$

$|z_3| = 4$

$$\frac{z_2}{z_3} = \frac{(2-2i)(2\sqrt{3}+2i)}{(2\sqrt{3}-2i)(2\sqrt{3}+2i)} = \frac{4\sqrt{3}+4i-4\sqrt{3}i+4}{16}$$

$$= \frac{4(\sqrt{3}+1)}{16} + i\,\frac{4(1-\sqrt{3})}{16} = \frac{\sqrt{3}+1}{16} + i\,\frac{1-\sqrt{3}}{16}$$

(III) $$(z_5)^8 = \left(\frac{1}{2}\cos\frac{\pi}{4} + i\frac{1}{2}\,\mathrm{sen}\,\frac{\pi}{4}\right)^8 =$$

De Moivre

$$= \frac{1}{2^8}\cos\left(\frac{8\pi}{4}\right) + i\,\frac{1}{2^8}\,\mathrm{sen}\left(\frac{8\pi}{4}\right) =$$

$$= \frac{1}{256}\cos(2\pi) + i\,\frac{1}{256}\,\mathrm{sen}(2\pi) = \frac{1}{256}$$

$z_2 + \overline{z}_2 = 2\,\mathrm{Re}(z_2)$

(IV) $$(z_2 + \overline{z}_2) \cdot z_3 \cdot z_4 = 4 \cdot 4_{-30^\circ} \cdot \sqrt{2}_{90^\circ} = 16\sqrt{2}_{60^\circ} =$$

$z_3 = 4_{-30^\circ}$

$z_4 = \sqrt{2}_{90^\circ}$

$= 16\sqrt{2}\cdot\left(\cos 60^\circ + i \operatorname{sen} 60^\circ\right) = 16\sqrt{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) =$

$= 8\sqrt{2} + 8\sqrt{6}\, i$

b) (I) $\left(\bar{z}_2\right)^{115} = \left(2\sqrt{2}_{315^\circ}\right)^{115}$

$|z_2| = 2\sqrt{2}$

$w_k = \sqrt[5]{2\sqrt{2}}_{\frac{315^\circ + 360^\circ k}{5}} = \sqrt[10]{8}_{63^\circ + 72^\circ k}$

$w_0 = \sqrt[10]{8}_{63^\circ}$, $w_1 = \sqrt[10]{8}_{135^\circ}$ $w_2 = \sqrt[10]{8}_{207^\circ}$

$w_3 = \sqrt[10]{8}_{279^\circ}$ $w_4 = \sqrt[10]{8}_{351^\circ}$

(II) $\sqrt[4]{z_3} = \sqrt[4]{4_{330^\circ}} = \sqrt[4]{4}_{\frac{330^\circ + 360^\circ k}{4}} =$

$= \sqrt{2}_{82'5^\circ + 90^\circ k}$

$w_0 = \sqrt{2}_{82'5^\circ}$ $w_1 = \sqrt{2}_{172'5^\circ}$

$w_2 = \sqrt{2}_{262'5^\circ}$ $w_3 = \sqrt{2}_{352'5^\circ}$

(III) $\sqrt[3]{\dfrac{z_1 - i}{z_3}} = \sqrt[3]{\dfrac{1+i}{2\sqrt{3} - 2i}} = \sqrt[3]{\dfrac{\sqrt{2}_{45^\circ}}{4_{-30^\circ}}}$

$= \sqrt[3]{\left(\frac{\sqrt{2}}{4}\right)_{-75^\circ}} = \sqrt[6]{\frac{1}{8}}_{\frac{-75^\circ + 360^\circ k}{3}} = \sqrt{\frac{1}{2}}_{-25^\circ + 120^\circ k} =$

$$= \left(\frac{\sqrt{2}}{2}\right)_{-25^\circ+120^\circ k}$$

$$w_0 = \left(\frac{\sqrt{2}}{2}\right)_{-25^\circ} \quad w_1 = \left(\frac{\sqrt{2}}{2}\right)_{95^\circ} \quad w_2 = \left(\frac{\sqrt{2}}{2}\right)_{215^\circ}$$

c) Conjugado de z_2: $\overline{z_2} = 2\sqrt{2}_{45^\circ}$

Opuesto de z_2:

$$-z_2 = -2\sqrt{2}_{-45^\circ} = 1_{180^\circ} \cdot 2\sqrt{2}_{-45^\circ} = 2\sqrt{2}_{135^\circ}$$

Inverso de z_2:

$$z_2^{-1} = \left(2\sqrt{2}_{-45^\circ}\right)^{-1} = (2\sqrt{2})^{-1}_{45^\circ} = \left(\frac{1}{2\sqrt{2}}\right)_{45^\circ} =$$

$$= \left(\frac{\sqrt{2}}{4}\right)_{45^\circ}$$

$\boxed{3}$ El complejo w es una de las raíces cúbicas de la unidad.
Las otras se obtienen multiplicando sucesivamente por $1_{120^\circ} = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

Si $w_0 = 2 - i4\sqrt{3}$:

$$w_1 = w_0 \cdot 1_{120^\circ} = (2 - i4\sqrt{3})\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) =$$

$$= -1 + \sqrt{3}i + 2\sqrt{3}i + 6 = 5 + 3\sqrt{3}i$$

$W_2 = W_1 \cdot 1_{120^\circ} = (5+3\sqrt{3}i)\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) =$

$= -\frac{5}{2} + \frac{5\sqrt{3}}{2}i - \frac{3\sqrt{3}}{2}i - \frac{9}{2} = -7+\sqrt{3}i$

El radio de dicha circunferencia es el módulo de cualquiera de las raíces:

$|W_2| = \sqrt{(-7)^2+(\sqrt{3})^2} = \sqrt{52}$ u

El lado del triángulo es

$|W_2 - W_1| = |-7+\sqrt{3}i - (5+3\sqrt{3}i)| =$

$= |-12-2\sqrt{3}i| = \sqrt{144+12} = \sqrt{156}$

Perímetro $= 3\sqrt{156}$ u

$\boxed{4}$ Dichos afijos son los vértices de un cuadrado centrado en el origen. Se obtienen multiplicando sucesivamente por $1_{90^\circ} = i$

$W_0 = 3-5i$

$W_1 = iW_0 = 5+3i$

$W_2 = iW_1 = -3+5i$

$W_3 = iW_2 = -5-3i$

Área $= |W_1 - W_0|^2 = |5+3i-(3-5i)|^2 = |2+8i|^2 =$

$= 20$ u^2