BOLETÍN LÍMITES

Exercicio 1.

a) $\lim_{x \to 0} \dfrac{3}{x-2} = \dfrac{3}{0-2} = -\dfrac{3}{2}$

b) $\lim_{x \to 3} \dfrac{1}{x^2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

c) $\lim_{x \to 3} \dfrac{4}{x^2+2x+3} = \dfrac{4}{3^2+2.3+3} = \dfrac{4}{9+6+3} = \dfrac{4}{18} = \dfrac{2}{9}$

d) $\lim_{x \to \infty} 2x^2-3x+5 = \infty$

e) $\lim_{x \to -\infty} 3x^2+5x-2 = \infty$

f) $\lim_{x \to \infty} -3x^2+5x-2 = -\infty$

g) $\lim_{x \to \infty} 2^x = 2^{\infty} = \infty$

h) $\lim_{x \to -\infty} \left(\dfrac{1}{2}\right)^x = \left(\dfrac{1}{2}\right)^{-\infty} = 2^{\infty} = \infty$

Exercicio 2.

a) $\lim\limits_{x\to 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$

b) $\lim\limits_{x\to -2} \frac{x-2}{x+2} = \frac{-2-2}{-2+2} = \frac{-4}{0}$

$$\begin{cases} \lim\limits_{x\to -2} \frac{x-2}{x+2} = \frac{-}{-}\infty = +\infty \\ \qquad \nexists \lim\limits_{x\to -2} \frac{x-2}{x+2} \\ \lim\limits_{x\to -2} \frac{x-2}{x+2} = \frac{-}{+}\infty = -\infty \end{cases}$$

c) $\lim\limits_{x\to 3} \frac{1}{x^2-9} = \frac{1}{3^2-9} = \frac{1}{9-9} = \frac{1}{0}$

$$\begin{cases} \lim\limits_{x\to 3^-} \frac{1}{x^2-9} = \frac{+}{-}\infty = -\infty \\ \qquad \nexists \lim\limits_{x\to 3} \frac{1}{x^2-9} \\ \lim\limits_{x\to 3^+} \frac{1}{x^2-9} = \frac{+}{+}\infty = +\infty \end{cases}$$

d) $\lim\limits_{x\to -1} \frac{x-3}{x+1} = \frac{-1-3}{-1+1} = \frac{-4}{0}$

$$\begin{cases} \lim\limits_{x\to -1^-} \frac{x-3}{x+1} = \frac{-}{-}\infty = +\infty \\ \qquad \nexists \lim\limits_{x\to -1} \frac{x-3}{x+1} \\ \lim\limits_{x\to -1^+} \frac{x-3}{x+1} = \frac{-}{+}\infty = -\infty \end{cases}$$

e) $\lim\limits_{x\to 1} \frac{1}{(x-1)^2} = \frac{1}{(1-1)^2} = \frac{1}{0}$

$$\begin{cases} \lim\limits_{x\to 1^-} \frac{1}{(x-1)^2} = \frac{+}{+}\infty = +\infty \\ \qquad \nexists \lim\limits_{x\to 1} \frac{1}{(x-1)^2} \\ \lim\limits_{x\to 1^+} \frac{1}{(x-1)^2} = \frac{+}{+}\infty = +\infty \end{cases}$$

Recordar que existen autores que se os límites laterales dan $+\infty$ os dous consideran que existe o límite.

f) $\lim\limits_{x\to 1} \frac{1}{(x-1)^3} = \frac{1}{(1-1)^3} = \frac{1}{0}$

$$\begin{cases} \lim\limits_{x\to 1^-} \frac{1}{(x-1)^3} = \frac{+}{-}\infty = -\infty \\ \qquad \nexists \lim\limits_{x\to 1} \frac{1}{(x-1)^3} \\ \lim\limits_{x\to 1^+} \frac{1}{(x-1)^3} = \frac{+}{+}\infty = +\infty \end{cases}$$

Exercicio 3.

a) $\lim_{x\to 2} \frac{x^2+x-6}{x^2-x-2} = \frac{2^2+2-6}{2^2-2-2} = \frac{6-6}{4-4} = \frac{0}{0}$ (indeterminación)

$$\lim_{x\to 2} \frac{x^2+x-6}{x^2-x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}(x+1)} = \lim_{x\to 2} \frac{x+3}{x+1} = \frac{2+3}{2+1} = \frac{5}{3}$$

Cálculos adicionais

$x^2+x-6=0$

$$x = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1\pm 5}{2} = \begin{cases} x_1 = \frac{-1+5}{2} = 2 \\ x_2 = \frac{-1-5}{2} = -3 \end{cases}$$

$x^2+x-6 = (x-2)(x+3)$

$x^2-x-2=0$

$$x = \frac{1\pm\sqrt{1+8}}{2} = \frac{1\pm 3}{2} = \begin{cases} x_1 = 2 \\ x_2 = -1 \end{cases}$$

$x^2-x-2 = (x-2)(x+1)$

b) $\lim\limits_{x\to 3} \dfrac{x^2-6x+9}{x-3} = \dfrac{3^2-6\cdot 3+9}{3-3} = \dfrac{9-18+9}{3-3} = \dfrac{0}{0}$ (indeterminación)

$$\lim_{x\to 3} \frac{x^2-6x+9}{x-3} = \lim_{x\to 3} \frac{(x-3)^2}{x-3} = \lim_{x\to 3} (x-3) = 3-3 = 0$$

c) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x^3-1} = \dfrac{1-1}{1-1} = \dfrac{0}{0}$ (indeterminación)

$$\lim_{x\to 1} \frac{x^2-1}{x^3-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x^2+x+1)} = \lim_{x\to 1} \frac{x+1}{x^2+x+1} =$$

$$= \frac{1+1}{1+1+1} = \frac{2}{3}$$

Cálculos adicionais

$x^2-1=0 \longrightarrow x^2=1 \longrightarrow x=\sqrt{1}=\pm 1 \Longrightarrow x^2-1=(x-1)(x+1)$

$x^3-1=0$ $\qquad\qquad$ $x^3-1=(x-1)(x^2+x+1)$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

d) $\lim_{x\to 2} \dfrac{-2x^2+5x-2}{3x^2-2x-8} = \dfrac{-2\cdot2^2+5\cdot2-2}{3\cdot2^2-2\cdot2-8} = \dfrac{-8+10-2}{12-4-8} = \dfrac{0}{0}$

(indeterminación)

$$\lim_{x\to 2} \frac{-2x^2+5x-2}{3x^2-2x-8} = \lim_{x\to 2} \frac{-2(x-\frac{1}{2})\cancel{(x-2)}}{3\cancel{(x-2)}(x+\frac{4}{3})} = \lim_{x\to 2} \frac{-2(x-\frac{1}{2})}{3(x+\frac{4}{3})} =$$

$$= \lim_{x\to 2} \frac{-2(x-1/2)}{3(x+\frac{4}{3})} = \frac{-2\cdot(2-\frac{1}{2})}{3(2+\frac{4}{3})} = \frac{-2\left(\frac{4-1}{2}\right)}{3\left(\frac{6+4}{3}\right)} = \frac{-\cancel{2}\cdot\frac{3}{\cancel{2}}}{\cancel{3}\cdot\frac{10}{\cancel{3}}} =$$

$$= -\frac{3}{10}$$

Cálculos adicionales

$$-2x^2+5x-2 = -2(x-\tfrac{1}{2})(x-2)$$

$$x = \frac{-5\pm\sqrt{25-16}}{-4} = \frac{-5\pm 3}{-4} = \begin{cases} x_1 = \dfrac{-5+3}{-4} = \dfrac{-2}{-4} = \dfrac{1}{2} \\ x_2 = \dfrac{-5-3}{-4} = \dfrac{-8}{-4} = 2 \end{cases}$$

$$3x^2-2x-8 = 3(x-2)(x+\tfrac{4}{3})$$

$$x = \frac{2\pm\sqrt{4+96}}{6} = \frac{2\pm 10}{6} = \begin{cases} x_1 = \dfrac{2+10}{6} = \dfrac{12}{6} = 2 \\ x_2 = \dfrac{2-10}{6} = -\dfrac{8}{6} = -\dfrac{4}{3} \end{cases}$$

e) $\lim\limits_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3} = \dfrac{\sqrt{3+1}-2}{3-3} = \dfrac{2-2}{3-3} = \dfrac{0}{0}$ (indeterminación)

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)} =$$

$$= \lim_{x\to 3} \frac{(\sqrt{x+1})^2-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)} =$$

$$= \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{2+2} = \frac{1}{4}$$

f) $\lim\limits_{x\to 0} \dfrac{\sqrt{2x+4}-2}{\sqrt{x+1}-1} = \dfrac{\sqrt{2\cdot 0+4}-2}{\sqrt{0+1}-1} = \dfrac{2-2}{1-1} = \dfrac{0}{0}$ (indeterminación)

$$\lim_{x\to 0} \frac{\sqrt{2x+4}-2}{\sqrt{x+1}-1} = \lim_{x\to 0} \frac{(\sqrt{2x+4}-2)(\sqrt{x+1}+1)}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)} =$$

$$= \lim_{x\to 0} \frac{(\sqrt{2x+4}-2)(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1^2} = \lim_{x\to 0} \frac{(\sqrt{2x+4}-2)(\sqrt{x+1}+1)}{x+\cancel{1}-\cancel{1}} =$$

$$= \lim_{x\to 0} \frac{(\sqrt{2x+4}-2)(\sqrt{x+1}+1)(\sqrt{2x+4}+2)}{x(\sqrt{2x+4}+2)} =$$

$$= \lim_{x\to 0} \frac{((\sqrt{2x+4})^2-2^2)(\sqrt{x+1}+1)}{x(\sqrt{2x+4}+2)} = \lim_{x\to 0} \frac{(2x+\cancel{4}-\cancel{4})(\sqrt{x+1}+1)}{x(\sqrt{2x+4}+2)} =$$

Continuación exercicio 3 f)

$$\lim_{x\to 0} \frac{2\cancel{x}\cdot(\sqrt{x+1}+1)}{\cancel{x}(\sqrt{2x+4}+2)} = \frac{2\cdot(\sqrt{0+1}+1)}{\sqrt{2\cdot 0+4}+2} = \frac{2\cdot(1+1)}{2+2} = \frac{4}{4}=1$$

Exercicio 4.

a) $\lim\limits_{x\to\infty} \dfrac{4x^2+x-1}{x^2+1} = \dfrac{\infty}{\infty}$ (Indeterminación)

$$\lim_{x\to\infty} \frac{4x^2+x-1}{x^2+1} = \lim_{x\to\infty} \frac{\frac{4x^2}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} =$$

$$= \lim_{x\to\infty} \frac{4+\frac{1}{x}-\frac{1}{x^2}}{1+1/x^2} = \frac{4+0-0}{1+0} = 4$$

b) $\lim\limits_{x\to\infty} \dfrac{x^4\ 3x^2+2x-1}{5x^3+3x-1} = \dfrac{\infty}{\infty}$ (Indeterminación)

$$\lim_{x\to\infty} \frac{x^4-3x^2+x-1}{5x^3+3x-1} = \lim_{x\to\infty} \frac{x^4/x^4-\frac{3x^2}{x^4}+\frac{x}{x^4}\ \frac{1}{x^4}}{\frac{5x^3}{x^4}+\frac{3x}{x^4}-\frac{1}{x^4}} =$$

$$= \lim_{x\to\infty} \frac{1-\frac{3}{x^2}+\frac{1}{x}-\frac{1}{x^4}}{\frac{5}{x}+\frac{3}{x^3}-\frac{1}{x^4}} = \frac{1-0+0-0}{0+0-0} = \frac{1}{0} = +\infty$$

c) $\lim\limits_{x\to\infty} \dfrac{4x+3}{5x^2-2x+5} = \dfrac{\infty}{\infty}$ (Indeterminación)

$$\lim_{x\to\infty} \frac{4x+3}{5x^2-2x+5} = \lim_{x\to\infty} \frac{4x/x^2+3/x^2}{\frac{5x^2}{x^2}-\frac{2x}{x^2}+\frac{5}{x^2}} = \lim_{x\to\infty} \frac{4/x+3/x^2}{5-\frac{2}{x}+\frac{5}{x}} =$$

$$= \frac{0+0}{5-0+0} = \frac{0}{5} = 0$$

d) $\lim\limits_{x\to\infty} \dfrac{-3x^2+5x-1}{2x^2+4x-1} = \dfrac{\infty}{\infty}$ (Indeterminación)

$$\lim_{x\to\infty} \frac{-3x^2+5x-1}{2x^2+4x-1} = \lim_{x\to\infty} \frac{\frac{-3x^2}{x^2}+\frac{5x}{x^2}-\frac{1}{x^2}}{\frac{2x^2}{x^2}+\frac{4x}{x^2}-\frac{1}{x^2}} =$$

$$= \lim_{x\to\infty} \frac{-3+\frac{5}{x}-\frac{1}{x^2}}{2+\frac{4}{x}-\frac{1}{x^2}} = \frac{-3+0-0}{2+0-0} = -\frac{3}{2}$$

e) $\lim\limits_{x\to\infty} \dfrac{5x^2-3x^2+1}{x-3} = \infty/\infty$ (Indeterminación)

$$\lim_{x\to\infty} \frac{5x^2-3x^2+1}{x-3} = \lim_{x\to\infty} \frac{2x^2+1}{x-3} = \lim_{x\to\infty} \frac{2x^2/x^2+1/x^2}{x/x^2-3/x^2} =$$

$$= \lim_{x\to\infty} \frac{2+1/x^2}{1/x-3/x^2} = \frac{2}{0} = +\infty$$

f) $\lim\limits_{x\to\infty} \dfrac{-4x^3+6x^2+5x}{6x^3+4x+3} = \dfrac{\infty}{\infty}$ (Indeterminación)

$$\lim_{x\to\infty} \frac{-4x^3+6x^2+5x}{6x^3+4x+3} = \lim_{x\to\infty} \frac{-4x^3/x^3+6x^2/x^3+5x/x^3}{6x^3/x^3+\frac{4x}{x^3}+3/x^3} =$$

$$= \lim_{x\to\infty} \frac{-4+6/x+5/x^2}{6+4/x^2+3/x^3} = \frac{-4}{6} = -\frac{2}{3}$$

g) $\lim\limits_{x\to\infty} \dfrac{-3x^3-2x^2+1}{4x^5+3x^2+2} = \infty/\infty$ (Indeterminación)

$$\lim_{x\to\infty} \frac{-3x^3-2x^2+1}{4x^5+3x^2+2} = \lim_{x\to\infty} \frac{-3x^3/x^5-\frac{2x^2}{x^5}+\frac{1}{x^5}}{\frac{4x^5}{x^5}+\frac{3x^2}{x^5}+\frac{2}{x^5}} = \frac{0}{4} = 0$$

h) $\lim\limits_{x\to\infty} \dfrac{-3x^5-2x^2+1}{4x^4+3x^2+2} = \infty/\infty$ (Indeterminación)

$$\lim_{x\to\infty} \frac{-3x^5-2x^2+1}{4x^4+3x^2+2} = \lim_{x\to\infty} \frac{-3x^5/x^5-2x^2/x^5+1/x^5}{\frac{4x^4}{x^5}+\frac{3x^2}{x^5}+\frac{2}{x^5}} = \lim_{x\to\infty} \frac{-3-2/x^3+1/x^5}{4/x+3/x^3+2/x^5}$$

$$= -\frac{3}{0} = -\infty$$

Exercicio 5.

a) $\lim_{x\to\infty}\left(\frac{x^2-1}{x}-\frac{1+2x^2}{2x-1}\right) = \infty - \infty$ (Indeterminación)

$$\lim_{x\to\infty}\left(\frac{x^2-1}{x}-\frac{1+2x^2}{2x-1}\right) = \lim_{x\to\infty}\left(\frac{(x^2-1)(2x-1)-x(1+2x^2)}{x(2x-1)}\right) =$$

$$= \lim_{x\to\infty}\frac{\cancel{2x^3}-x^2-2x+1-x-\cancel{2x^3}}{2x^2-2x} = \lim_{x\to\infty}\frac{-x^2-3x+1}{2x^2-2x} = \frac{\infty}{\infty}$$

(Indeterminación).

$$\lim_{x\to\infty}\frac{-x^2-3x+1}{2x^2-2x} = \lim_{x\to\infty}\frac{-\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{1}{x^2}}{\frac{2x^2}{x^2}-\frac{2x}{x^2}} =$$

$$= \lim_{x\to\infty}\frac{-1-\frac{3}{x}+\frac{1}{x^2}}{2-\frac{2}{x}} = \frac{-1-0+0}{2-0} = -\frac{1}{2}$$

b) $\lim_{x\to\infty} \left(2x - \sqrt{1+4x}\right) = \infty - \infty$ (Indeterminación)

$$\lim_{x\to\infty} \left(2x - \sqrt{1+4x}\right) = \lim_{x\to\infty} \left(\frac{\left(2x - \sqrt{1+4x}\right)\cdot\left(2x+\sqrt{1+4x}\right)}{2x+\sqrt{1+4x}}\right) =$$

$$= \lim_{x\to\infty} \frac{(2x)^2 - \left(\sqrt{1+4x}\right)^2}{2x+\sqrt{1+4x}} = \lim_{x\to\infty} \frac{4x^2 - (1+4x)}{2x+\sqrt{1+4x}} =$$

$$= \lim_{x\to\infty} \frac{4x^2-4x-1}{2x+\sqrt{1+4x}} = \frac{\infty}{\infty} \quad \text{(Indeterminación)}$$

$$\lim_{x\to\infty} \frac{4x^2-4x-1}{2x+\sqrt{1+4x}} = \lim_{x\to\infty} \frac{\frac{4x^2}{x^2} - \frac{4x}{x^2} - \frac{1}{x^2}}{\frac{2x}{x^2} + \sqrt{\frac{1}{x^4} + \frac{4x}{x^4}}} =$$

$$= \lim_{x\to\infty} \frac{4 - \frac{4}{x} - \frac{1}{x^2}}{\frac{2}{x} + \sqrt{\frac{1}{x^4} + \frac{4}{x^3}}} = \frac{+4}{0} = +\infty$$

c) $\lim_{x\to\infty} \sqrt{x^2-2x} - x = \infty - \infty$ (Indeterminación)

$$\lim_{x\to\infty} \sqrt{x^2-2x} - x = \lim_{x\to\infty} \frac{\left(\sqrt{x^2-2x}-x\right)\left(\sqrt{x^2-2x}+x\right)}{\sqrt{x^2-2x}+x} =$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2-2x}\right)^2 - x^2}{\sqrt{x^2-2x}+x} = \lim_{x\to\infty} \frac{\cancel{x^2}-2x-\cancel{x^2}}{\sqrt{x^2-2x}+x} =$$

$$= \lim_{x\to\infty} \frac{-2x}{\sqrt{x^2-2x}+x} = \frac{\infty}{\infty} \quad \text{(Indeterminación)}$$

$$\lim_{x\to\infty} \frac{-2x}{\sqrt{x^2-2x}+x} = \lim_{x\to\infty} \frac{\frac{-2x}{x}}{\sqrt{\frac{x^2}{x^2}-\frac{2x}{x^2}}+\frac{x}{x}} =$$

$$= \lim_{x\to\infty} \frac{-2}{\sqrt{1-\frac{2}{x}}+1} = \frac{-2}{\sqrt{1-0}+1} = \frac{-2}{\sqrt{1}+1} =$$

$$= \frac{-2}{1+1} = \frac{-2}{2} = -1$$

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d) $\lim_{x\to\infty} \sqrt{9x^2+3x-3} - 3x \stackrel{?}{=} \infty - \infty$ (Indeterminación)

$$\lim_{x\to\infty} \sqrt{9x^2+3x-3} - 3x = \lim_{x\to\infty} \frac{\left(\sqrt{9x^2+3x-3} - 3x\right)\left(\sqrt{9x^2+3x-3}+3x\right)}{\sqrt{9x^2+3x-3}+3x} =$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{9x^2+3x-3}\right)^2 - (3x)^2}{\sqrt{9x^2+3x-3}+3x} = \lim_{x\to\infty} \frac{9x^2+3x-3-9x^2}{\sqrt{9x^2+3x-3}+3x} =$$

$$= \lim_{x\to\infty} \frac{3x-3}{\sqrt{9x^2+3x-3}+3x} = \frac{\infty}{\infty}$$ (Indeterminación)

$$\lim_{x\to\infty} \frac{3x-3}{\sqrt{9x^2+3x-3}+3x} = \lim_{x\to\infty} \frac{\frac{3x}{x} - \frac{3}{x}}{\sqrt{\frac{9x^2}{x^2}+\frac{3x}{x^2}-\frac{3}{x^2}} + \frac{3x}{x}} =$$

$$= \lim_{x\to\infty} \frac{3 - \frac{3}{x}}{\sqrt{9+\frac{3}{x}-\frac{3}{x^2}}+3} = \frac{3-0}{\sqrt{9}+3} = \frac{3}{3+3} = \frac{3}{6} = \frac{1}{2}$$

Exercicio 6.

a) $\lim_{x\to\infty}\left(\frac{x+5}{x-3}\right)^{x+2} = 1^{\infty}$ (Indeterminación)

$$\lim_{x\to\infty}\left(\frac{x+5}{x-3}\right)^{x+2} = \lim_{x\to\infty}\left(1+\frac{x+5}{x-3}-1\right)^{x+2} =$$

$$= \lim_{x\to\infty}\left(1+\frac{x+5-(x-3)}{x-3}\right)^{x+2} = \lim_{x\to\infty}\left(1+\frac{\cancel{x}+5\cancel{-x}+3}{x-3}\right)^{x+2} =$$

$$= \lim_{x\to\infty}\left(1+\frac{8}{x-3}\right)^{x+2} = \lim_{x\to\infty}\left(\left(1+\frac{1}{\frac{x-3}{8}}\right)^{\frac{x-3}{8}}\right)^{\left(\frac{8}{x-3}\right)(x+2)} =$$

$$= e^{\lim_{x\to\infty}\frac{8}{(x-3)}(x+2)} = e^{\lim_{x\to\infty}\frac{8x+16}{x-3}} =$$

$$= e^{\lim_{x\to\infty}\frac{\frac{8x}{x}+\frac{16}{x}}{\frac{x}{x}-\frac{3}{x}}} = e^{\frac{8+0}{1-0}} = e^{8}$$

b) $\lim_{x\to\infty}\left(\frac{7x^2+1}{7x^2-13}\right)^{3x^2+2} = 1^{\infty}$ (Indeterminación)

$$\lim_{x\to\infty}\left(\frac{7x^2+1}{7x^2-13}\right)^{3x^2+2} = \lim_{x\to\infty}\left(1+\frac{7x^2+1}{7x^2-13}-1\right)^{3x^2+2} =$$

$$= \lim_{x\to\infty}\left(1+\frac{7x^2+1-(7x^2-13)}{7x^2-13}\right)^{3x^2+2} =$$

$$= \lim_{x\to\infty}\left(1+\frac{\cancel{7x^2}+1-\cancel{7x^2}+13}{7x^2-13}\right)^{3x^2+2} = \lim_{x\to\infty}\left(1+\frac{14}{7x^2-13}\right)^{3x^2+2} =$$

$$= \lim_{x\to\infty}\left(\left(1+\frac{1}{\frac{7x^2-13}{14}}\right)^{\frac{7x^2-13}{14}}\right)^{\left(\frac{14}{7x^2-13}\right)(3x^2+2)} =$$

$$= e^{\lim_{x\to\infty}\frac{14(3x^2+2)}{7x^2-13}} = e^{\lim_{x\to\infty}\frac{42x^2+28}{7x^2-13}} =$$

$$= e^{\lim_{x\to\infty}\frac{42x^2/x^2+28/x^2}{7x^2/x^2-13/x^2}} = e^{\lim_{x\to\infty}\frac{42+28/x^2}{7-13/x^2}} = e^{\frac{42+0}{7-0}} = e^6$$

d) $\lim\limits_{x \to \infty} \left(\dfrac{x^2+2x-1}{x^2} \right)^{\frac{x^2+1}{x-1}} = 1^{\infty}$ (Indeterminación)

$$\lim_{x \to \infty} \left(\frac{x^2+2x-1}{x^2} \right)^{\frac{x^2+1}{x-1}} = \lim_{x \to \infty} \left(1 + \frac{x^2+2x-1}{x^2} - 1 \right)^{\frac{x^2+1}{x-1}} =$$

$$= \lim_{x \to \infty} \left(1 + \frac{\cancel{x^2}+2x-1-\cancel{x^2}}{x^2} \right)^{\frac{x^2+1}{x-1}} = \lim_{x \to \infty} \left(1 + \frac{2x-1}{x^2} \right)^{\frac{x^2+1}{x-1}} =$$

$$= \lim_{x \to \infty} \left(1 + \frac{1}{\frac{x^2}{2x-1}} \right)^{\frac{x^2+1}{x-1}} = \lim_{x \to \infty} \left(\left(1 + \frac{1}{\frac{x^2}{2x-1}} \right)^{\frac{x^2}{2x-1}} \right)^{\frac{2x-1}{x^2} \cdot \frac{x^2+1}{x-1}} =$$

$$= e^{\lim\limits_{x \to \infty} \frac{(2x-1)(x^2+1)}{x^2(x-1)}} = e^{\lim\limits_{x \to \infty} \frac{2x^3+2x-x^2-1}{x^3-x^2}} =$$

$$= e^{\lim\limits_{x \to \infty} \frac{2\frac{x^3}{x^3}+2\frac{x}{x^3}-\frac{x^2}{x^3}-\frac{1}{x^3}}{\frac{x^3}{x^3}-\frac{x^2}{x^3}}} = e^{\lim\limits_{x \to \infty} \frac{2+\frac{2}{x^2}-\frac{1}{x}-\frac{1}{x^3}}{1-1/x}} =$$

$$= e^{\frac{2+0-0-0}{1-0}} = e^2$$

d) $\lim_{x\to\infty} \left(\frac{5x+1}{5x-1}\right)^{3x+2} = 1^{\infty}$ (Indeterminación)

$$\lim_{x\to\infty} \left(\frac{5x+1}{5x-1}\right)^{3x+2} = \lim_{x\to\infty} \left(1 + \frac{5x+1}{5x-1} - 1\right)^{3x+2} =$$

$$= \lim_{x\to\infty} \left(1 + \frac{5x+1-(5x-1)}{5x-1}\right)^{3x+2} =$$

$$= \lim_{x\to\infty} \left(1 + \frac{\cancel{5x}+1-\cancel{5x}+1}{5x-1}\right)^{3x+2} = \lim_{x\to\infty} \left(1 + \frac{2}{5x-1}\right)^{3x+2} =$$

$$= \lim_{x\to\infty} \left(\left(1 + \frac{1}{\frac{5x-1}{2}}\right)^{\frac{5x-1}{2}}\right)^{\left(\frac{2}{5x-1}\right)(3x+2)} =$$

$$= e^{\lim_{x\to\infty} \frac{6x+4}{5x-1}} = e^{\lim_{x\to\infty} \frac{6x/x + 4/x}{5x/x - 1/x}} =$$

$$= e^{\lim_{x\to\infty} \frac{6+4/x}{5-1/x}} = e^{\frac{6+0}{5-0}} = e^{6/5}$$

e) $\lim_{x\to\infty}\left(\frac{x^3+2x+1}{x^3-1}\right)^{\frac{x+1}{x-1}} = 1^\infty$ (Indeterminación)

$$\lim_{x\to\infty}\left(\frac{x^3+2x+1}{x^3-1}\right)^{\frac{x+1}{x-1}} = \lim_{x\to\infty}\left(1+\frac{x^3+2x+1}{x^3-1}-1\right)^{\frac{x+1}{x-1}} =$$

$$= \lim_{x\to\infty}\left(1+\frac{\cancel{x^3}+2x+1-\cancel{x^3}+1}{x^3-1}\right)^{\frac{x+1}{x-1}} = \lim_{x\to\infty}\left(1+\frac{2x+2}{x^3-1}\right)^{\frac{x+1}{x-1}} =$$

$$= \lim_{x\to\infty}\left(1+\frac{1}{\frac{x^3-1}{2x+2}}\right)^{\frac{x+1}{x-1}} = \lim_{x\to\infty}\left(\left(1+\frac{1}{\frac{x^3-1}{2x+2}}\right)^{\frac{x^3-1}{2x+2}}\right)^{\left(\frac{2x+2}{x^3-1}\right)\left(\frac{x+1}{x-1}\right)} =$$

$$= e^{\lim_{x\to\infty}\left(\frac{2x+2}{x^3-1}\right)\left(\cdot\frac{x+1}{x-1}\right)} = e^{\lim_{x\to\infty}\frac{2x^2+2x+2x+2}{x^4-x^3-x+1}} =$$

$$= e^{\lim_{x\to\infty}\frac{2x^2+4x+2}{x^4-x^3-x+1}} = e^{\lim_{x\to\infty}\frac{2x^2/x^4+\frac{4x}{x^4}+2/x^4}{\frac{x^4}{x^4}-\frac{x^3}{x^4}-\frac{x}{x^4}+1/x^4}} =$$

$$= e^{\frac{0+0+0}{1-0-0+1}} = e^0 = 1$$

f) $\lim\limits_{x \to \infty} \left(\frac{x^2-1}{x^3-3x+1} \right)^{\frac{x+1}{x-1}} = 0^1 = 0$