MATHEMATICS

YEAR 4

SECOND TERM

LESSON 4: **Trigonometry**
LESSON 5: **Vectors and Coordinate Geometry**
LESSON 6: **Geometry and Transformations**

Measure of angles

A **degree** can be divided into smaller parts, called minutes.
There are 60 minutes in a degree. We write $60' = 1°$.
Similarly, $½° = 30'$ and $¼ = 15'$. Also, $3.1° = 3.1 \times 60 = 186'$.
A minute can be subdivided into 60 seconds, but these are so small that we do not bother with them.

To enter degrees and minutes into a calculator, look for the button marked (which stands for Degrees, Minutes, Seconds). This button is sometimes labelled .

To write 24°33′ on a calculator, press 2 4 … 3 3

Example 1

Two angles of a triangle have measure 53°48′ and 77°31′. Find the measure of the third angle.

Solution

Press:

1 8 0 DMS − 5 3 DMS 4 8 DMS − 7 7 DMS 3 1 DMS =

The answer 48°41′ appears on the screen.

One **radian** is the angle subtended at the center of a circle by an arc of circumference that is equal in length to the radius of the circle.

Angles can also be measured in **radians** and this makes it much easier to deal w… trigonometric functions when using **calculus**.

1 radian ≈ 57.3°. This may be written as $1^c \approx 57.3°$. However, the symbol … radians is not normally written when the angle involves π.
The following results are useful to remember:

$\pi = 180°, \frac{\pi}{2} = 90°, \frac{\pi}{4} = 45°, \frac{\pi}{3} = 60°, \frac{\pi}{6} = 30°.$

... thing is that you know how radians relate to degrees.

π rad = 180° | 2π rad = 360°

1 rad = $\frac{180°}{\pi} \approx 57.3°$ | $\frac{\pi}{2}$ rad = 90°

You've got to know these ones well — so you don't need to think about them at all

To do a really rough check of an answer, take π as 3, so $90° \approx \frac{3}{\ldots}$ radians, and a whole circle's about 6 radians. Always make sure the answer looks about right.

...ou need to convert from one to the other:

Radians to Degrees

... π, multiply by 180.

Degrees to Radians

Divide by 180, multiply by π.

You can easily work this out, since you know that π rads is 180°

The **grad** is a unit of plane angle, equivalent to 1⁄400 of a full circle, dividing a right angle in 100. It is also known as **gon**, **grade**, or **gradian**. The unit was really only adopted in some countries and for specialized areas, like land measurement.
Note: 1 grad of a great circle course on the surface of the Earth corresponds to 100 km distance.

How do you convert degrees, minutes, and seconds to and from a decimal number?
Many calculators have a built-in function to compute this - it is often called "dms" or "hms".
Examples:

(from decimal to degrees, minutes and seconds)	**Write 42.36824** Click the "DMS" button 42° 22' 5.66'' will appear
(from degrees, minutes and seconds to decimal)	Write 35 Click the "DMS" button Write 48 Click the "DMS" button, (35.8 will appear) Write 50 Click the "DMS" button 35.813889° will appear

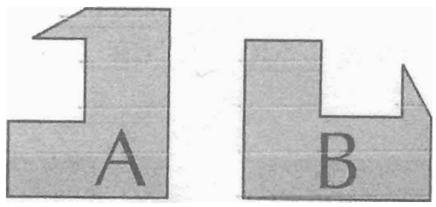
Similarity and enlargement

Congruence and Similarity

Congruence is another ridiculous maths word which sounds really complicated when it's not. If two shapes are congruent, they're simply the same — the same size and the same shape. That's all it is.

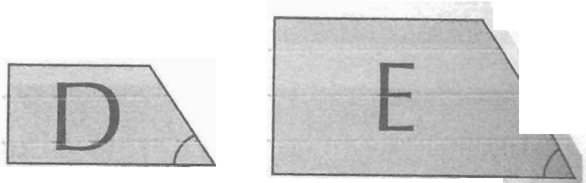
Congruent

— same size, same shape: A, B, and C are congruent (with each other).



Similar

— same shape, different size: D and E are similar (but they're not congruent).

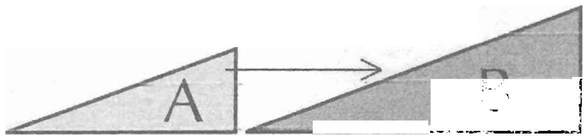


For shapes to be similar, the angles must be the same.

Enlargements

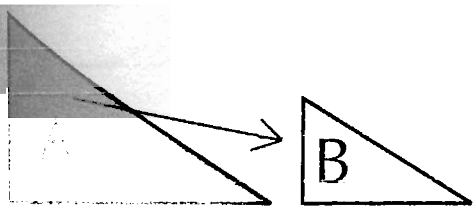
... are four main things that you need to know about enlargements:

1 If the **scale factor** is bigger than 1 then the shape gets bigger.



A to B is an Enlargement, Scale Factor 1½

2

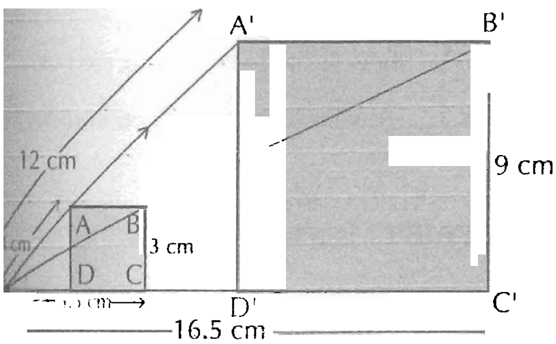


... B is an Enlargement of Scale Factor ½

If the scale factor is smaller than 1 (i.e. a fraction like ½), then the shape gets smaller.

(Really this is a reduction, but you still call it an enlargement, Scale Factor ½)

3 Enlargement Scale Factor 3



The scale factor also tells you the relative distance of old points and new points from the centre of enlargement.

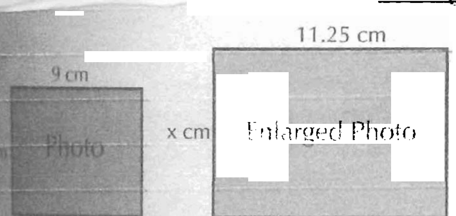
This is very useful for drawing an enlargement, because you can use it to trace out the positions of the new points from the centre of enlargement, as shown in the diagram.

ENLARGEMENT

4 ... lengths of the two shapes (big and small) are ... to the scale factor by this very important ...

NEW LENGTH = SCALE FACTOR × OLD LENGTH

Classic "enlarged photo" exam question with no trouble:



To find the width of the enlarged photo we use the formula triangle twice, firstly to find the scale factor, and then to find the missing side:

1) S.F. = new length ÷ old length = 11.25 ÷ 9 = 1.25

2) New width = scale factor × old width = 1.25 × 6.4 = 8 cm

For an enlargement of scale factor n:

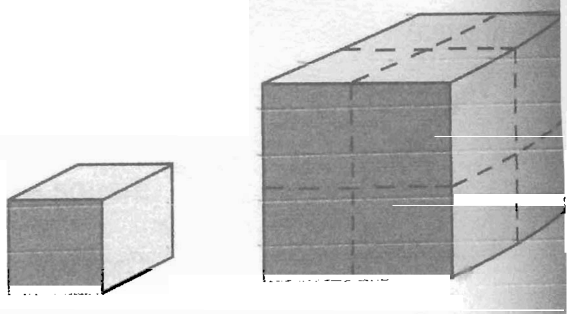
The SIDES are n times bigger
The AREAS are n^2 times bigger
The VOLUMES are n^3 times bigger

Simple... but very forgettable.

For example, if the scale factor is 2:

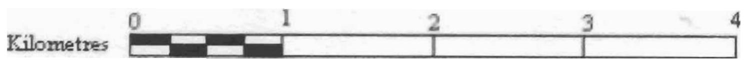
1) the lengths are twice as big, $(n = 2)$
2) each area is 4 times as big, $(n^2 = 4)$
3) the volume is 8 times as big, $(n^3 = 8)$
as shown in the diagram:

All you have to do is remember it.



Map scale is the relationship between a unit of length on a map and the corresponding length over the ground

A typical verbal scale might be "One inch to one mile". Many maps carry a graphic scale such as this bar scale.



A **representative fraction (RF)** shows the relationship between one of any unit on the map and the same units on the ground. RFs may be shown as an actual fraction (e.g. 1/25,000). They are more usually written like a mathematical proportion with a colon (as in 1:25,000).

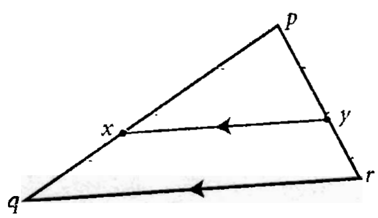
Example: 1:25,000 means that one unit of any length on the map represents 25,000 of the same units on the ground

The RF is the "scale factor" of the supposed enlargement between the map and the ground.
LENGTH ON THE MAP = LENGTH OVER THE GROUND · RF

Similarity of triangles

Theorem: ***A line drawn parallel to one side of a triangle divides the other two sides in the same ratio.***

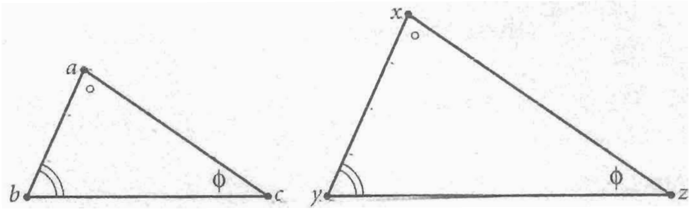
i.e. $|px| : |xq| = |py| : |yr|$



The converse of this theorem is true. That is, if $\frac{|px|}{|xq|} = \frac{|py|}{|yr|}$, then $xy \parallel qr$.

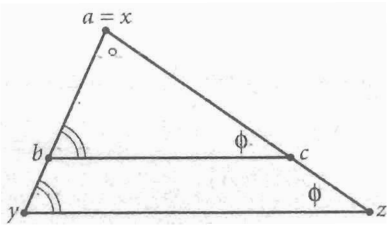
(Thales) *Theorem:* ***If two triangles are equiangular, the lengths of corresponding sides are proportional.***

Given: Two triangles $\triangle abc$ and $\triangle xyz$ whose corresponding angles are equal in measure.



To prove: $\frac{|ab|}{|xy|} = \frac{|ac|}{|xz|} = \frac{|bc|}{|yz|}$

Construction: Move $\triangle xyz$ so that $a = x$ and b is on $[xy]$. c will be on $[xz]$ because $|\angle bac| = |\angle yxz|$.



Proof: $bc \parallel yz$, since corresponding angles $\angle abc$ and $\angle xyz$ are equal.

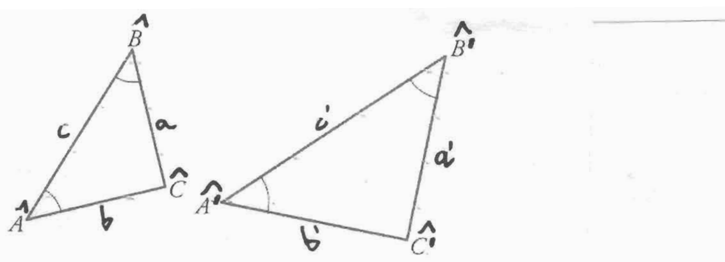
$\therefore \frac{|ab|}{|xy|} = \frac{|ac|}{|xz|}$ (as a consequence of Theorem J)

Similarly, it can be proved that $\frac{|ab|}{|xy|} = \frac{|bc|}{|yz|}$

$\therefore \frac{|ab|}{|xy|} = \frac{|ac|}{|xz|} = \frac{|bc|}{|yz|}$ QED

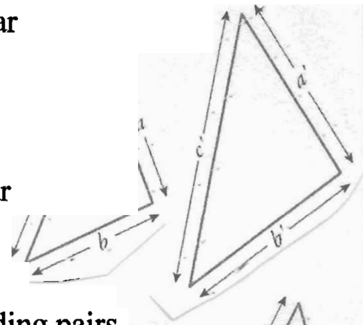
<u>Comsequences</u>:

(Let's suppose that $\hat{A}$, $\hat{B}$, $\hat{C}$ are the angles of a triangle and *a, b, c* are the lengths of the respectively opposite sides of the triangle; the same for $\hat{A}'$, $\hat{B}'$, $\hat{C}'$ and *a', b', c'*).



Angle-Angle (AA) theorem
If two triangles have two pairs of corresponding angles equal, then the triangles are similar.

$$\hat{A} = \hat{A}' \text{ and } \hat{B} = \hat{B}' \Rightarrow \widehat{ABC} \text{ and } \widehat{A'B'C'} \text{ are similar}$$

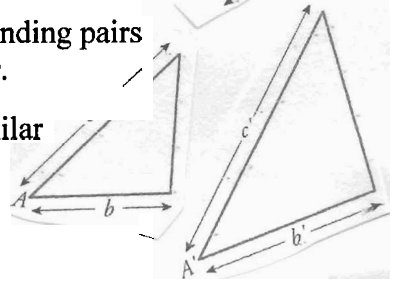


Side-Side-Side (SSS) theorem
If two triangles have all three pairs of corresponding sides in the same ratio, then the triangles are similar.

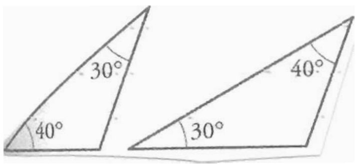
$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c} \Rightarrow \widehat{ABC} \text{ and } \widehat{A'B'C'} \text{ are similar}$$

Side-Angle-Side (SAS) theorem
If two triangles have one pair of corresponding angles equal and both corresponding pairs of sides adjacent to the angle have the same ratio, then the triangles are similar.

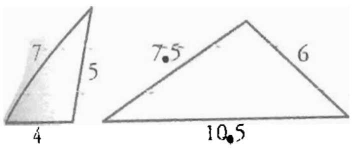
$$\hat{A} = \hat{A}' \text{ and } \frac{b'}{b} = \frac{c'}{c} \Rightarrow \widehat{ABC} \text{ and } \widehat{A'B'C'} \text{ are similar}$$



Examples:

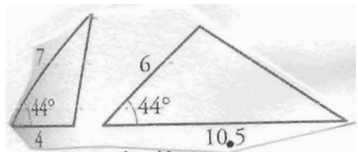


Are similar



Are similar

$$\frac{10.5}{7} = \frac{6}{4} = \frac{7.5}{5} = 1.5$$

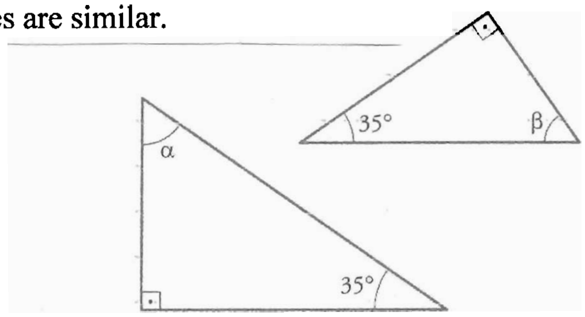


Are similar

$$\frac{7}{10.5} = \frac{4}{6} = 0.\hat{6}$$

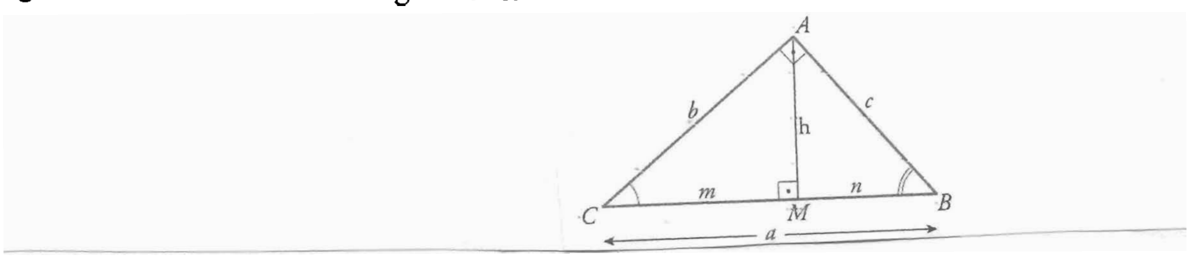
Further comsequences:

- When two right-angled triangles have one pair of corresponding acute angles equal the triangles are similar.



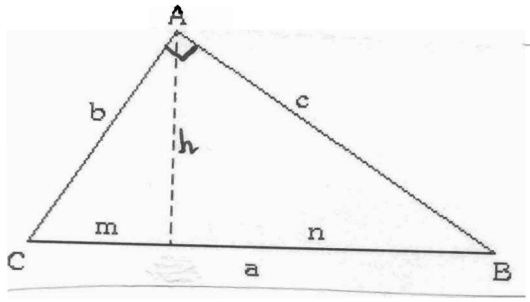
$$\left.\begin{array}{l} 90° + 35° + \alpha = 180° \\ 90° + 35° + \beta = 180° \end{array}\right\} \rightarrow \alpha = \beta$$

- The height perpendicular to the hypothenuse of a right-angled triangle splits it into two triangles that are similar to the original one.



Theorems in right-angled triangles

Let "a", "b" and "c" be the hypothenuse and the short-sides of a right-angled triangle; let "m" be the vertical projection of "b" onto the hypothenuse and "n" the vertical projection of "c" onto the hypothenuse; leth "h" be the height drawn from the hypothenuse. Thus, the intersection of "h" and the hypothenuse "a" divides "a" in two parts that are "m" and "n".



The perpendicular height theorem:
In a right-angled triangle the height drawn from the hypotenuse is the geometric mean of the two parts that it divides the hypotenuse into: $h = \sqrt{m \cdot n}$. This can be written $\boxed{h^2 = m \cdot n}$

Prove: height "h" divides the triangle into two triangles
they are both right-angled triangles and the sides of the two acute angles are perpendicular to each other
so their angles are equal
so the triangles are similar
so their ratios of the short sides are equal: $\frac{h}{m} = \frac{n}{h}$

The theorem of the sides adjacent to the right angle
In a right-angled triangle each of the short-sides is the geometric mean of its projection onto the hypotenuse and the hypotenuse itself: $b = \sqrt{m \cdot a}$ and $c = \sqrt{n \cdot a}$. These can be written $\boxed{b^2 = m \cdot a}$ and $\boxed{c^2 = n \cdot a}$

Prove: height "h" splits the triangle into two triangles
they are both right-angled triangles and they share an acute angle with the big triangle
so their angles are equal to the big triangle ones
so the small triangles are similar to the big triangle
so their ratios of the hypothenuse and one short side are equal: $\frac{b}{m} = \frac{a}{b}$ and $\frac{c}{n} = \frac{a}{c}$

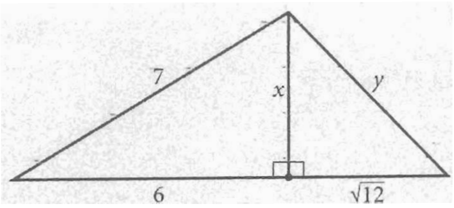
The Pythagoras' theorem:
In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. That can be written $\boxed{a^2 = b^2 + c^2}$

Prove: applying the previous theorem: $b^2 + c^2 = m \cdot a + n \cdot a = (m + n) \cdot a = a \cdot a = a^2$

Note: the converse is true ($a^2 = b^2 + c^2 \Rightarrow$ the triangle has a right angle between the sides of lengths b and c)

Example 1

Find x and y, as in the diagram:



Solution

$x^2 + 6^2 = 7^2$ (by Pythagoras' Theorem)

$\therefore\ x^2 + 36 = 49$

$\therefore\ x^2 = 13$

$\therefore\ x = \sqrt{13}$ (leave surds as surds)

Now, $x^2 + (\sqrt{12})^2 = y^2$

$\therefore\ (\sqrt{13})^2 + (\sqrt{12})^2 = y^2$

$\therefore\ 13 + 12 = y^2$

$\therefore\ 25 = y^2$

$\therefore\ 5 = y$

Answer $x = \sqrt{13};\ y = 5$

Example 2

The lengths of sides of a triangle are 28, 45 and 53. Investigate if the triangle is right-angled.

Solution

We must investigate if $53^2 = 45^2 + 28^2$

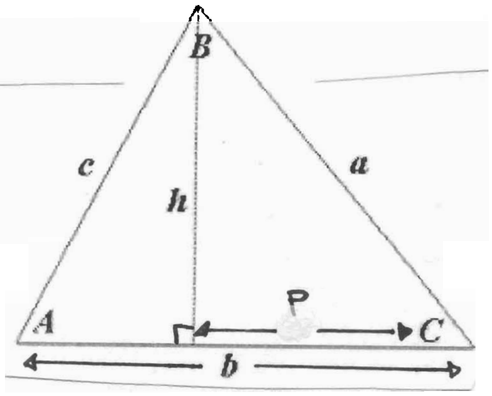
$53^2 = 2809;\ 45^2 + 28^2 = 2025 + 784 = 2809$

Since $53^2 = 45^2 + 28^2$, we can conclude that the triangle *is* right-angled, by the converse of Pythagoras' theorem.

Generalized Pythagoras' theorem

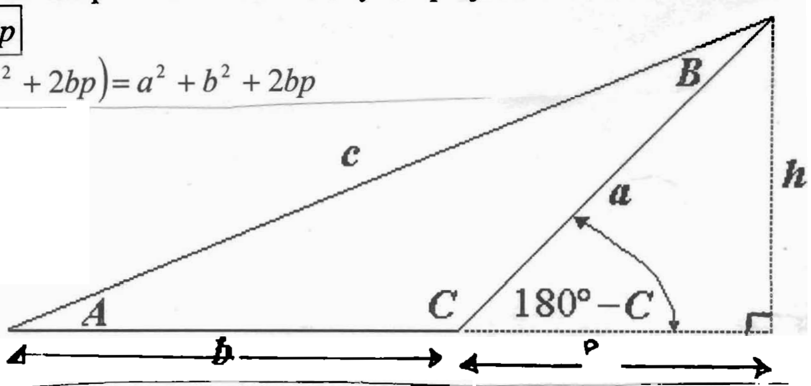
In acute-angled triangles the square on the side opposite the acute angle is is equal to the the sum of the squares on the sides containing the obtuse angle minus twice the product of the base by the segment of base out of the projection of the other side onto the base $\boxed{c^2 = a^2 + b^2 - 2bp}$

Prove: $c^2 = h^2 + (b-p)^2 = (a^2 - p^2) + (b^2 + p^2 - 2bp) = a^2 + b^2 - 2bp$



In obtuse-angled triangles the square on the side opposite the obtuse angle is equal to the the sum of the squares on the sides containing the obtuse angle plus twice the product of the base by the projection of the other side onto the base's prolongation $\boxed{c^2 = a^2 + b^2 + 2bp}$

Prove: $c^2 = h^2 + (b+p)^2 = (a^2 - p^2) + (b^2 + p^2 + 2bp) = a^2 + b^2 + 2bp$



EXERCISES

1) How could you state 40° 50’ 20’’ as an angle using common decimal notation?

2) Can we express 40.3472° in units of degrees, minutes, and seconds?

3) How could you state 100° 28’ 55’’ as an angle using common decimal notation?

4) Can we express 8.4816° in units of degrees, minutes, and seconds?

5) How many minutes in each of the following?
(i) half a degree
(ii) ¼ of a degree
(iii) ⅓ of a degree
(iv) 1½°
(v) ⅒ of a degree
(vi) 2.5°
(vii) 1.75°
(viii) 10⅔°
(ix) 3¾°
(x) 4.2°

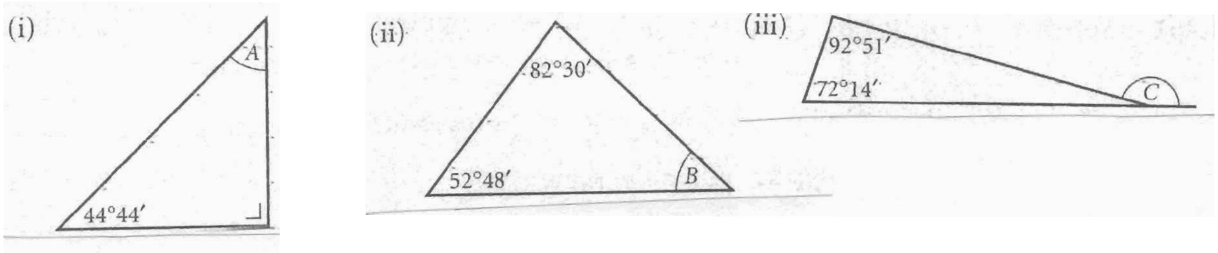
6) Perform the following additions (and subtractions), giving your answer in degrees and minutes:
(i) 2°30′ + 4°15′
(ii) 6°12′ + 10°40′
(iii) 25°50′ + 42°20′
(iv) 61°55′ + 70°22′
(v) 11°37′ + 24°38′
(vi) 41°41′ + 48°19′
(vii) 7°52′ + 10°8′
(viii) 19°50′ – 6°10′
(ix) 10°10′ – 2°40′
(x) 90° – 33°33′

7) a) Two angles of a triangle are of measure 63°25′ and 51°45′. Find the third angle.

b) Two angles of a triangle are of measure 44°22′ and 85°57′. Find the third angle.

c) Two angles of a triangle are *both* of measure 55°42′. Find the third angle.

8) Find the missing angles *A, B, C, D, E*:



9) Express in radians:

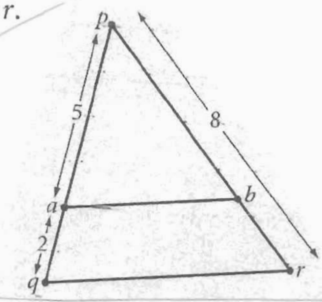
330°	135°	210°	315°	120°	270°
225°	330°	150°	240°	300°	100°

10) Express in degrees:

$\frac{2\pi}{3}$ *rad*	$\frac{5\pi}{6}$ *rad*	$\frac{3\pi}{4}$ *rad*	$\frac{7\pi}{6}$ *rad*	$\frac{5\pi}{4}$ *rad*
$\frac{4\pi}{3}$ *rad*	$\frac{3\pi}{2}$ *rad*	$\frac{11\pi}{6}$ *rad*	$\frac{5\pi}{3}$ *rad*	$\frac{7\pi}{4}$ *rad*

11) In the diagram, $ab \parallel qr$.

Find $|br|$.



12) Investigate if triangles with these lengths of sides are right-angled, or not:

(i) 8, 15, 17
(ii) 7, 11, 13
(iii) $\sqrt{2}, \sqrt{3}, \sqrt{5}$
(iv) 7, 24, 25
(v) 20, 99, 101
(vi) 3, $\sqrt{7}$, 4

13) What is the area of an isosceles triangle with equal sides 5 cm long and different side 6 cm long?

14) Find out the perpendicular height drawn from the hypothenuse ("h") and the projections of the short sides onto the hypothenuse ("m", "n").
Data: the hypothenuse "a" is 5 m long; the short-sides "b" and "c" are 3 and 4 m long respectively.

15) Find out the perpendicular height drawn from the hypothenuse ("h"), the short-side ("c"), its projection onto the hypothenuse ("n") and the hypothenuse ("a").
Data: the short-side "b" is 16.5 cm long and its projection onto the hypothenuse "m" is 7.5 cm long.

16) Find out the perpendicular height drawn from the hypothenuse (“h”), the short-side (“b”), its projection onto the hypothenuse (“m”) and the hypothenuse (“a”).
Data: the short-side “c” is 70 cm and its projection onto the hypothenuse “n” is 50 cm long.

17) Find out the short-side (“c”), the height drawn from the hypothenuse (“h”), the hypothenuse (“a”) and the projections of the short-sides onto it (“m”, “n”).
Data: the short-side “b” is 12 cm long and meets the hypothenuse at an angle of 60°.

Heron's formula

Heron's formula for the area of a triangle with sides of length a, b, c is

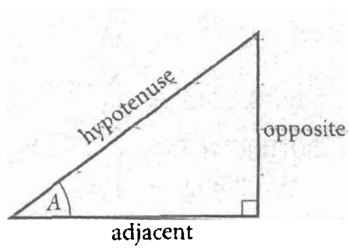
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$ where $$s = \frac{a+b+c}{2}$$

Sines, cosines and tangents

Here is a right-angled triangle, with an angle A. The longest side is called the **hypotenuse**, as we have seen.

The side opposite the angle A is called the **opposite**.

The third side is called the **adjacent** because it is adjacent to (or beside) the angle A.



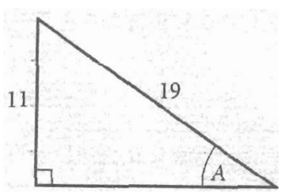
$$\text{sine } A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine } A = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{tangent } A = \frac{\text{opposite}}{\text{adjacent}}$$

(These names are sometimes shortened to **sin**, **cos** and **tan**.)

Example

Write the sine A as a fraction (see diagram).

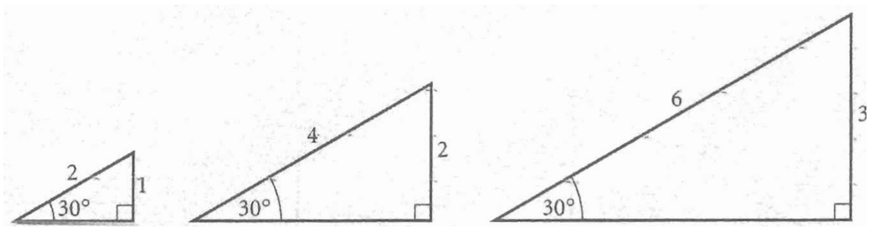


Solution

$$\text{sine } A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{11}{19}$$

The trigonometric ratios **do not depend upon the chosen triangle** where they are calculated.

Example: Here are three right-angled triangles. Each has an angle of 30°.



Did you notice anything? Even though the triangles are of different sizes, the length of the side opposite the 30° angle is always half of the length of the hypotenuse.

i.e. $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} = 0.5$ (Where 'opposite' means the side **opposite** the angle.)

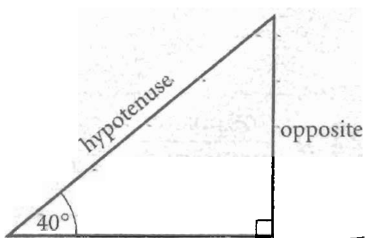
This ratio will hold for **any** right-angled triangle with an angle of 30°.
This ratio is known as the **sine** of the angle 30°.

On your calculator press

[3] [0] [=]

The answer 0.5 should appear on the screen. (If it doesn't, get someone to make sure that your calculator is in **degree mode.** Then try again.)

If you press [4] [0] [=] on your calculator, the number 0.642787609 appears on the screen. We usually shorten these numbers to four decimal places, which is accurate enough for most calculations.



$\therefore$ sine 40° = 0.6428

This means that in **any** triangle which has a right-angle and an angle of 40°, then

$\frac{\text{opposite}}{\text{hypotenuse}} = 0.6428$

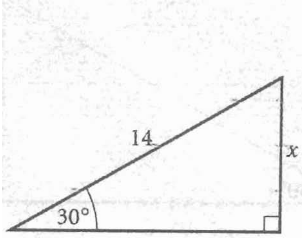
Supposing you want to find an angle whose sine is 0.8 (i.e. you want to find A if $\sin A = 0.8$).

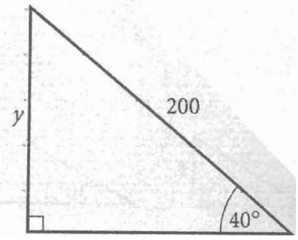
You must press [0] [.] [8] [=]

The answer is 53.1301°, which is 53° to the nearest degree.

Example 1

Find the values of x and y to the nearest whole number.





Solution

(i)

$$\sin 30° = 0.5$$
$$\therefore \frac{\text{opposite}}{\text{hypotenuse}} = 0.5$$
$$\therefore \frac{x}{14} = 0.5$$
$$\therefore x = 14(0.5)$$
$$\therefore x = 7$$

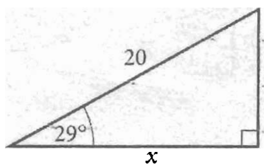
(ii)

$$\sin 40° = 0.6428$$
$$\therefore \frac{\text{opposite}}{\text{hypotenuse}} = 0.6428$$
$$\therefore \frac{y}{200} = 0.6428$$
$$\therefore y = 200(0.6428)$$
$$\therefore y = 128.56$$
$$\therefore y = 129$$

(to the nearest whole number)

Example 2

Find x correct to one decimal place.



Solution

The two sides with either a number or a letter are the **adjacent** and the **hypotenuse.** Therefore, we will use the ratio which involves these two sides only.

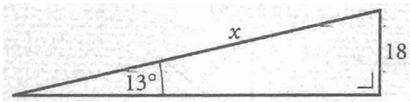
It is **cosine** $A = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$

$$\therefore \cos 29° = \frac{x}{20}$$
$$\therefore 0.8746 = \frac{x}{20}$$
$$\therefore 20(0.8746) = x$$
$$\therefore 17.492 = x$$
$$\therefore x = 17.5 \text{ (correct to one decimal place)}$$

Answer 17.5

Example 3

Find the value of x correct to the nearest whole number:



Solution

The side with a number or a letter are the **opposite** and the **hypotenuse**. We will therefore use the formula which mentions these two sides.

$$\text{It is } \textbf{sine } A = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$$

$$\therefore \sin 13° = \frac{18}{x}$$

$$\therefore 0.225 = \frac{18}{x}$$

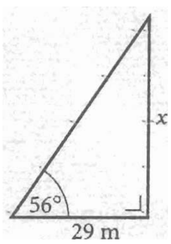
$$\therefore 0.225x = 18$$

$$\therefore x = \frac{18}{0.225}$$

$$\therefore x = 80$$

Answer 80

Example 4



Find x to the nearest metre.

Solution

The sides with a number or a letter are the **opposite** and the **adjacent**. Therefore we will use the formula which mentions these two sides.

$$\text{It is } \textbf{tangent } A = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\therefore \tan 56° = \frac{x}{29}$$

$$\therefore 1.4826 = \frac{x}{29}$$

$$\therefore 29(1.4826) = x$$

$$\therefore 42.9954 = x$$

$$\therefore x = 43 \text{ m} \quad \text{(to the nearest metre)}$$

Answer 43 m

Trigonometric identities

Learn these trigonometric identities; they are very important when simplifying expressions and solving equations and should be learnt.

$$\sin^2 x + \cos^2 x = 1$$

("Pythagorean trigonometric identity")

$$\frac{\sin x}{\cos x} = \tan x$$

$$1 + \tan^2 x = \frac{1}{\cos^2 x}$$

Prove of the 1st:

applying Pythagoras' theorem $\quad opposite^2 + adjacent^2 = hypothenuse^2$

dividing by $hypothenuse^2$ $\quad \frac{opposite^2}{hypothenuse^2} + \frac{adjacent^2}{hypothenuse^2} = \frac{hypothenuse^2}{hypothenuse^2}$

that can be written $\quad \left(\frac{opposite}{hypothenuse}\right)^2 + \left(\frac{adjacent}{hypothenuse}\right)^2 = 1$

so we have got $\quad \sin^2 x + \cos^2 x = 1$

Prove of the 2nd:

$$\frac{\sin x}{\cos x} = \frac{\frac{opposite}{hypothenuse}}{\frac{adjacent}{hypothenuse}} = \frac{opposite}{adjacent} = \tan x$$

Prove of the 3rd:

$$1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

Example

Solve the equation $\sin x = \sqrt{3} \cos x$, for values of x between 0° and 360°.

Divide each side by $\cos x$.

$$\frac{\sin x}{\cos x} = \sqrt{3}$$

$$\Rightarrow \tan x = \sqrt{3}$$

$$x = 60° \text{ or } 240°$$

Three special angles: 45°, 30° and 60°

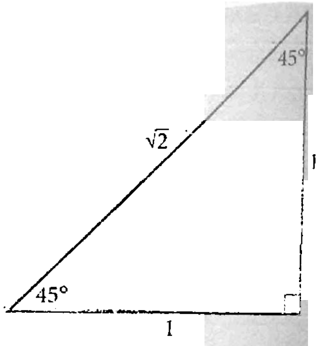
45°

Let h = the length of the hypotenuse.

By Pythagoras' theorem, $h^2 = 1^2 + 1^2$

$\therefore h^2 = 2$

$\therefore h = \sqrt{2}$



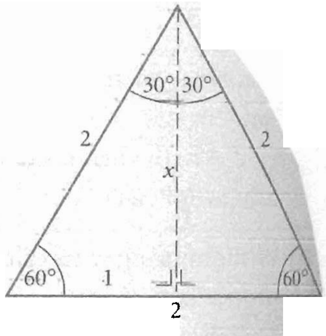
ISOSCELES

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} \quad \therefore \cos 45° = \dfrac{1}{\sqrt{2}}$

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} \quad \therefore \sin 45° = \dfrac{1}{\sqrt{2}}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}} \quad \therefore \tan 45° = \dfrac{1}{1} = 1$

30° AND 60°



EQUILATERAL

Since the triangle is equilateral, each angle is 60°.

Now if you bisect the triangle, you could get a right-angled triangle with other angles of 60° and 30°.

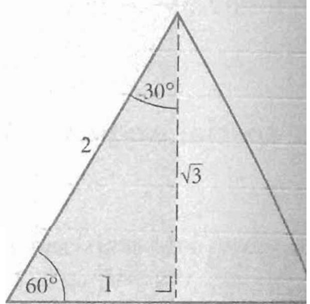
Let x = the side opposite 60°.

By Pythagoras' Theorem, $2^2 = 1^2 + x^2$

$\therefore 4 = 1 + x^2$

$\therefore 3 = x^2$

$\therefore x = \sqrt{3}$



$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} \quad \therefore \cos 60° = \dfrac{1}{2}$

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} \quad \therefore \sin 60° = \dfrac{\sqrt{3}}{2}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}} \quad \therefore \tan 60° = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} \quad \therefore \cos 30° = \dfrac{\sqrt{3}}{2}$

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} \quad \therefore \sin 30° = \dfrac{1}{2}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}} \quad \therefore \tan 30° = \dfrac{1}{\sqrt{3}}$

Using trigonometric identities in proves

Another use for these trig identities is proving that two things are the same.

Example 1 Show that $\frac{\cos^2\theta}{1+\sin\theta} \equiv 1-\sin\theta$

The identity sign ≡ means that this is true for all θ, rather than just certain values

Prove things like this by playing about with one side of the equation until it you get the other side.

Left-hand side: $\frac{\cos^2\theta}{1+\sin\theta}$

The only thing I can think of doing here is replacing $\cos^2\theta$ with $1-\sin^2\theta$. (Which is good because it works.)

$$\equiv \frac{1-\sin^2\theta}{1+\sin\theta}$$

The next trick is the hardest to spot. Look at the top — does that remind you of anything?

The top line is a difference of two squares:

$$\equiv \frac{(1+\sin\theta)(1-\sin\theta)}{1+\sin\theta}$$

$1-a^2=(1+a)(1-a)$

$\Rightarrow 1-\sin^2\theta=(1+\sin\theta)(1-\sin\theta)$

$\equiv 1-\sin\theta$, the right-hand side.

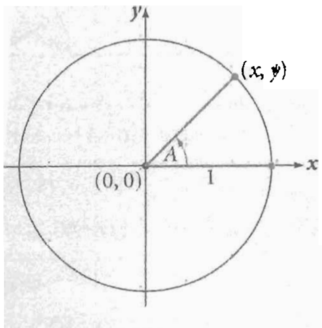
Secant, cosecant and cotangent

These trigonometric ratios, commonly known as sec, cosec and cot, are defined from the more familiar sin, cos and tan ratios as follows:

$$\sec x = \frac{1}{\cos x} \qquad \operatorname{cosec} x = \frac{1}{\sin x} \qquad \cot x = \frac{1}{\tan x}$$

Trigonometric ratios of non-acute angles

The definitions $\sin A = \frac{\text{opp}}{\text{hyp}}$, $\cos A = \frac{\text{adj}}{\text{hyp}}$, $\tan A = \frac{\text{opp}}{\text{adj}}$ are fine if A is an acute angle. But if A is obtuse (say 120°) then it is impossible to draw a right-angled triangle with angles 90° and 120°. New definitions are called for.

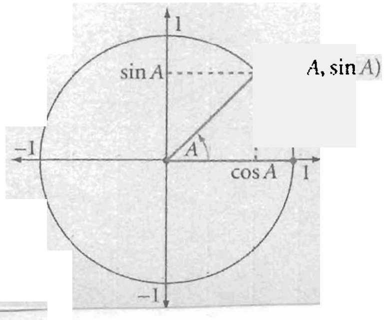


Draw a circle with centre (0, 0) and radius one unit in length (called a **unit circle**). Draw a radius along the positive sense of the x-axis. Now draw another radius, at an angle A to the first radius (turning anti-clockwise). If (x, y) is the point where this radius meets the circle, then

$$\cos A = x,\ \sin A = y,\ \tan A = \frac{y}{x} = \frac{\sin A}{\cos A}$$

remember that

[redacted] y) = (cos A, sin A)



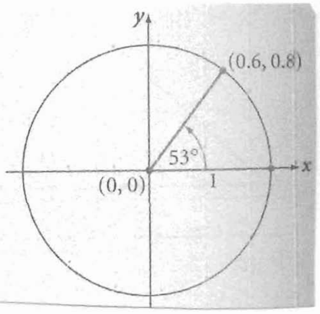
For example, when $A = 53°$, we find that the point

$(x, y) = (0.6, 0.8)$

$\therefore\ \cos 53° = 0.6$

$\sin 53° = 0.8$

$\tan 53° = \frac{0.8}{0.6} = \frac{4}{3}$



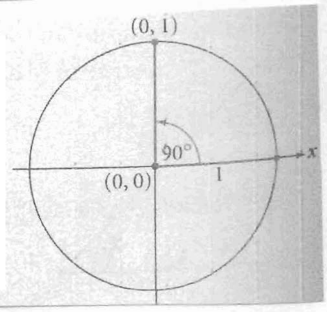
Also, if $A = 90°$,

the point $(x, y) = (0, 1)$

$\therefore\ \cos 90° = 0,\ \sin 90° = 1$

and $\tan 90° = \frac{1}{0}$ = undefined

(because you cannot divide by zero).



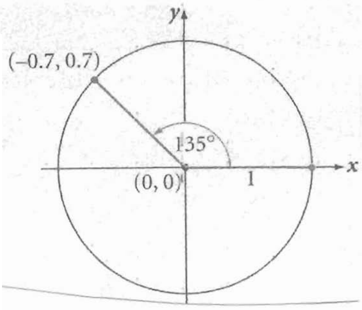
Now, if $A = 135°$, then

$(x, y) = (-0.7, 0.7)$

$\therefore\ \cos 135° = -0.7$

$\sin 135° = 0.7$

$\tan 135° = \frac{0.7}{-0.7} = -1$



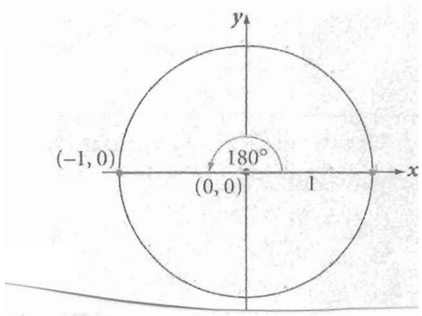
[redacted] = 180°, then

$(x, y) = (-1, 0)$

$\therefore\ \cos 180° = -1$

$\sin 180° = 0$

$\tan 180° = \frac{0}{-1} = 0$



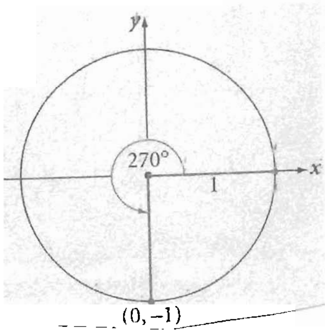
[redacted]nally, if $A = 270°$, then

$(x, y) = (0, -1)$

$\therefore\ \cos 270° = 0$

$\sin 270° = -1$

$\tan 270° = \frac{-1}{0}$ = undefined

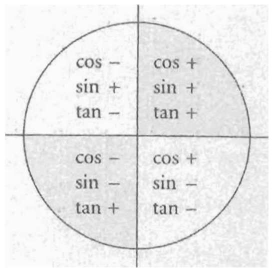


The four quadrants

Let us divide the unit circle into four quadrants.

- The 1st quadrant is for angles in the range: 0° to 90°.
- The 2nd quadrant is for angles in the range: 90° to 180°.
- The 3rd quadrant is for angles in the range: 180° to 270°.
- The 4th quadrant is for angles in the range: 270° to 360°.

In the 1st quadrant, all values of cos, sin and tan are positive.



In the 2nd quadrant, cosines are negative, sines are positive and tangents are negative.

(since $\tan = \frac{\sin}{\cos} = \frac{\text{positive}}{\text{negative}} = \text{negative}$)

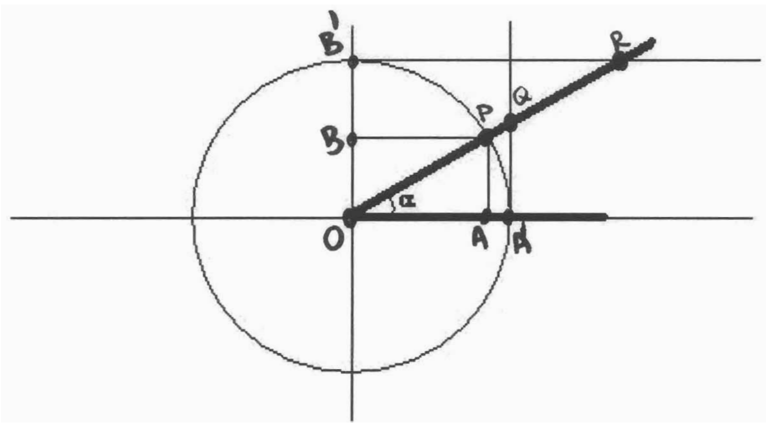
In the 3rd quadrant, cosines and sines are both negative. Hence tangents are positive.

(since $\tan = \frac{\sin}{\cos} = \frac{\text{negative}}{\text{negative}} = \text{positive}$)

In the 4th quadrant, cosines are positive, sines are negative and tangents are negative.

(since $\tan = \frac{\sin}{\cos} = \frac{\text{negative}}{\text{positive}} = \text{negative}$)

Geometric interpretation of the trigonometric ratios



$\sin\alpha = \overline{OB} = \overline{AP}$

$\cos\alpha = \overline{OA} = \overline{BP}$

$\tan\alpha = \overline{A'Q}$ because $\tan\alpha = \frac{\overline{AP}}{\overline{OA}} = \frac{\overline{A'Q}}{\overline{OA'}} = \frac{\overline{A'Q}}{1}$

$\cos ec\alpha = \overline{OR}$ because $\frac{1}{sen\alpha} = \frac{\overline{OP}}{\overline{OB}} = \frac{\overline{OR}}{\overline{OB'}} = \frac{\overline{OR}}{1}$

$\sec\alpha = \overline{OQ}$ because $\frac{1}{\cos\alpha} = \frac{\overline{OP}}{\overline{OA}} = \frac{\overline{OQ}}{\overline{OA'}} = \frac{\overline{OQ}}{1}$

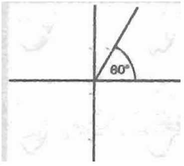
$\cot an\alpha = \overline{B'R}$ because $\cot an\alpha = \frac{\overline{BP}}{\overline{OB}} = \frac{\overline{B'R}}{\overline{OB'}} = \frac{\overline{B'R}}{1}$

Trigonometric equations

Example: Solve $\cos x = \frac{1}{2}$ for $-360° \le x \le 720°$.

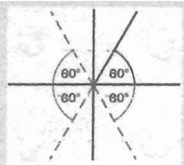
First, to find all the values of x between 0° and 360° where $\cos x = \frac{1}{2}$ — you do this:

Put the first solution onto the diagram.



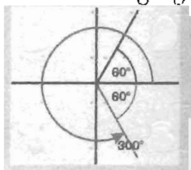
The angle from your calculator goes anticlockwise from the x-axis (unless it's negative — then it would go clockwise into the 4th quadrant).

Find the other angles between 0° and 360° that might be solutions.



The other solutions come from making the same angle from the horizontal axis into the other 3 quadrants.

Ditch the ones that are the wrong sign.



$\cos x = \frac{1}{2}$, which is positive. cos is positive in the 4th quadra[nt] not the 2nd or 3rd — so ditch those tw[o]

So you've got solutions 60° and 300° in the range 0° to 360°. But you need all the solutions in the range –360° to 720°. Get these by repeatedly adding or subtracting 360° onto each until you go out of range:

$x = 60° \Rightarrow$ (adding 360°) $x = 420°$, 780° (too big)
and (subtracting 360°) $x = -300°$, $-660°$ (too small)

$x = 300° \Rightarrow$ (adding 360°) $x = 660°$, 1020° (too big)
and (subtracting 360°) $x = -60$, $-420°$ (too small)

So the solutions are: $x = -300°, -60°, 60°, 300°, 420°$ and $660°$.

...if you have a $\sin^2 x$ or a $\cos^2 x$, think of this straight away...

$$\sin^2 x + \cos^2 x \equiv 1 \Rightarrow \begin{array}{l}\sin^2 x \equiv 1 - \cos^2 x \\ \cos^2 x \equiv 1 - \sin^2 x\end{array}$$

Use this identity to get rid of a $\sin^2$ or a $\cos^2$ that's making things awkward...

Example: Solve: $2\sin^2 x + 5\cos x = 4$, for $0° \le x \le 360°$.

You can't do much while the equation's got both sin's and cos's in it. So replace the $\sin^2 x$ bit with $1 - \cos^2 x$.

$$2(1-\cos^2 x) + 5\cos x = 4$$

Multiply out the bracket and rearrange it so that you've got zero on one side — and you get a quadratic in $\cos x$:

$$\Rightarrow 2 - 2\cos^2 x + 5\cos x = 4$$
$$\Rightarrow 2\cos^2 x - 5\cos x + 2 = 0$$

Now the only trig function is cos

If you replaced cos x with y, this would be $2y^2 - 5y + 2 = 0$

This is a quadratic in $\cos x$. It's easier to factorise this if you make the substitution $y = \cos x$.

$$2y^2 - 5y + 2 = 0$$
$$\Rightarrow (2y-1)(y-2) = 0$$
$$\Rightarrow (2\cos x - 1)(\cos x - 2) = 0$$

$2y^2 - 5y + 2 = (2y \ \ ?)(y \ \ ?)$
$= (2y-1)(y-2)$

Now one of the brackets must be 0. So you get 2 equations as usual:

$(2\cos x - 1) = 0$ or $(\cos x - 2) = 0$

$\cos x = \frac{1}{2} \Rightarrow x = 60°$ or $x = 300°$ and $\cos x = 2$

This is a bit weird. cos x is always between –1 and 1. So you don't get any solutions from this bracket.

This is impossible — so you get nothing from this bracket.

So at the end of all that, the only solutions you get are $x = 60°$ and $x = 300°$. How boring.

For equations with tan x in, it often helps to use this...

$$\tan x \equiv \frac{\sin x}{\cos x}$$

This is a handy thing to know — and one the examiners love testing. Basically, if you've got a trig equation with a tan in it, together with a sin or a cos — chances are you'll be better off if you rewrite the tan using this formula.

Example: Solve: $3\sin x - \tan x = 0$, for $0° \le x \le 360°$

It's got sin and tan in it — so writing tan x as $\frac{\sin x}{\cos x}$ is probably a good move:

$$3\sin x - \tan x = 0$$
$$\Rightarrow 3\sin x - \frac{\sin x}{\cos x} = 0$$

Get rid of the cos x on the bottom by multiplying the whole equation by cos x.

$$\Rightarrow 3\sin x\cos x - \sin x = 0$$

Now — there's a common factor of sin x. Take that outside a bracket.

$$\Rightarrow \sin x(3\cos x - 1) = 0$$

And now you're almost there. You've got two things multiplying together to make zero. That means either one or both of them is equal to zero themselves.

$$\Rightarrow \sin x = 0 \quad \text{or} \quad 3\cos x - 1 = 0$$

The first solution is... $\sin^{-1} 0 = 0°$

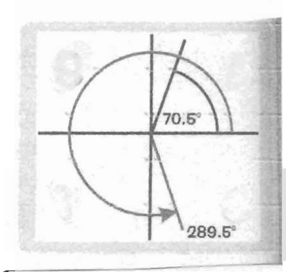
Now find the other points where sin x is zero in the interval $0° \le x \le 360°$.
(Remember the sin graph is zero every *180°*.)

$$\Rightarrow x = 0°, 180°, 360°$$

Rearrange... $\cos x = \frac{1}{3}$

So the first solution is...

$$\cos^{-1}\tfrac{1}{3} = 70.5°$$



. another solution in the 4th quadrant...

So altogether you've got five possible solutions:

$$\Rightarrow x = 0°, 180°, 360°, 70.5°, 289.5°$$

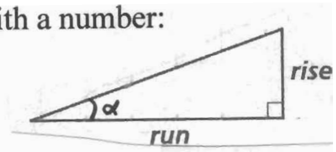
And the two solutions from this part are:

$$\Rightarrow x = 70.5°, 289.5°$$

Slopes and tangents

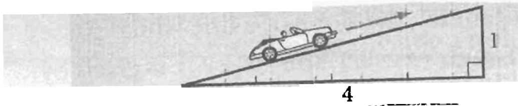
If the hill rises as we travel from left to right, we can express the slope with a number:

$$slope = \frac{rise}{run} = \tan\alpha$$

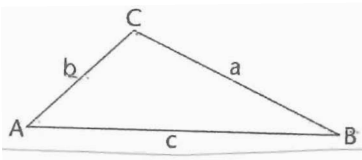


Sometimes it is given as a percenteage, previously multiplied by 100.

Example: Here is a hill whose slope is +¼ or 25 %



The sine and cosine rules

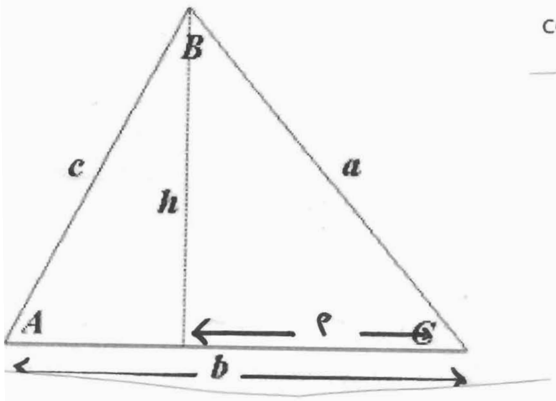


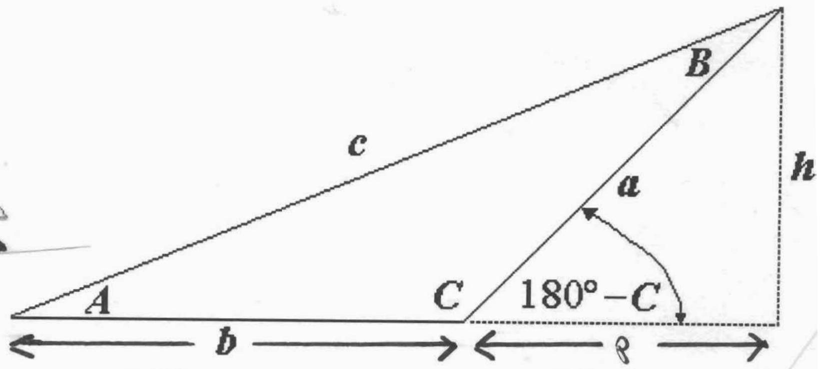
Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Note: This may be rearranged to find an angle

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$





Prove of the sine rule:

$$\left.\begin{matrix} \frac{h}{a} = \sin C \Rightarrow h = a\cdot\sin C \\ \frac{h}{c} = \sin A \Rightarrow h = c\cdot\sin A \end{matrix}\right\} \Rightarrow a\cdot\sin C = c\cdot\sin A \Rightarrow \frac{a}{\sin A} = \frac{c}{\sin C}$$

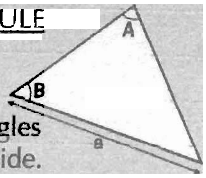
Prove of the cosine rule:

If C is obtuse $c^2 = h^2 + (b+p)^2 = (a^2 - p^2) + (b^2 + p^2 + 2bp) = a^2 + b^2 + 2ba(-\cos C) = a^2 + b^2 - 2ba\cos C$ because $\cos C$ is a negative number

If C is acute $c^2 = h^2 + (b-p)^2 = (a^2 - p^2) + (b^2 + p^2 - 2bp) = a^2 + b^2 - 2b\cos C$ because $\cos C$ is a positive number

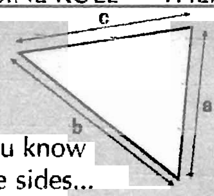
To decide which of these two rules you need to use, look at how much you already know about the triangle.

SINE RULE

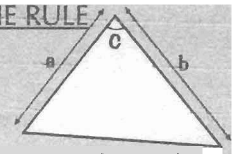


If you know two angles and a side.
(And if you know two angles, you can easily find the other one.)

COSINE RULE — THEN SINE RULE



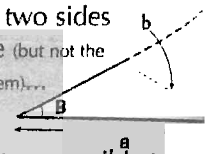
If you know three sides...



...or two sides and the angle between them.

ERR — IT DOESN'T WORK HE

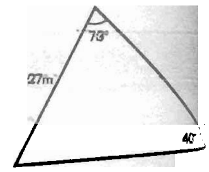
If you've got two sides and an angle (but not the angle between them)...



...there are sometimes two possible trian

Example

Solve ΔABC, in which A = 40°, a = 27m, B = 73°. Then find its area.



Draw a quick sketch first — don't worry if it's not deadly accurate, though.
You're given two angles and a side, so you need the Sine Rule.

Make sure you put side a opposite angle A.

First of all, get the other angle:

$$\angle C = (180 - 40 - 73)° = 67°$$

Then find the other sides, one at a time:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{27}{\sin 40°} = \frac{b}{\sin 73°}$$

$$\Rightarrow b = \frac{27}{\sin 40°} \times \sin 73° = 40.169 = \underline{40.2\text{m}}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow \frac{c}{\sin 67°} = \frac{27}{\sin 40°}$$

$$\Rightarrow c = \frac{27}{\sin 40°} \times \sin 67° = 38.665 = \underline{38.7}$$

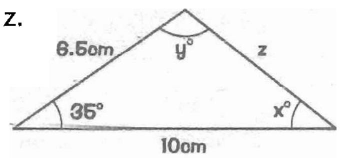
Now just use the formula to find its area. Area of Δ ABC $= \frac{1}{2}ab\sin C$

$$= \tfrac{1}{2} \times 27 \times 40.169 \times \sin 67°$$

$$= \underline{499.2\text{m}^2}$$

Example

Find X, Y and z.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

You've been given 2 sides and the angle between them, so you're going to need the Cosine Rule, then the Sine Rule.

$$z^2 = (6.5)^2 + 10^2 - 2(6.5)(10)\cos 35°$$

In this case, angle A is 35°, and side a is actually z

$$\Rightarrow z^2 = 142.25 - 130\cos 35°$$

$$z^2 = 35.7602$$

$$\Rightarrow \underline{z = 5.9800 = 5.98\text{cm}} \text{ (to 2 d.p.)}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

When you use an earlier answer, don't use one that's been rounded too much. Use at least 3 or 4 decimal places. So it's a good idea to write down a really exact answer to each part and then round it.

You've got all the sides. Now use the Sine Rule to find another two angles.

The Sine Rule just says: Dividing "length of a side" by "the sine of the opposite angle" gives the same answer, whichever pair you choose.

$$\frac{6.5}{\sin X} = \frac{5.9800}{\sin 35°}$$

$$\Rightarrow \sin X = 0.6234$$

$$\Rightarrow X = \sin^{-1} 0.6234$$

$$\Rightarrow \underline{X = 38.6°}$$

To get the last angle, just subtract the two angles you know from 180°:

35° + 38.6° + Y = 180°

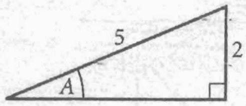
Y = 180 - 35 - 38.6 = <u>106.4°</u>

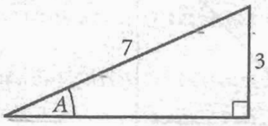
EXERCISES

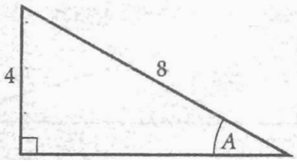
18) Find the sine of these angles, correct to four decimal places:

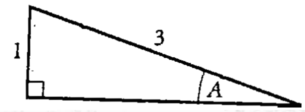
(i) 60° (ii) 80° (iii) 25°
(iv) 1° (v) 10° (vi) 34°
(vii) 48° (viii) 6° (ix) 54°
(x) 2°
(xi) 70° (xii) 15° (xiii) 8°
(xiv) 83° (xv) 77° (xvi) 41°
(xvii) 88° (xviii) 11° (xix) 23°
(xx) 59°

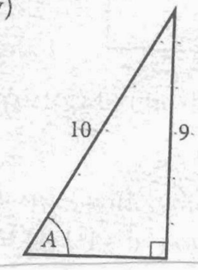
19) Write sine A as a fraction in each case:

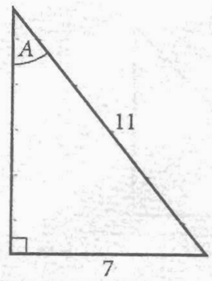
(i)


(ii)


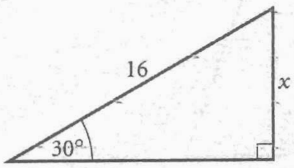
(iii)


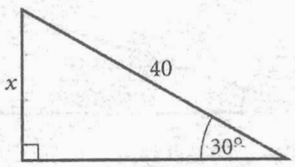
(iv)


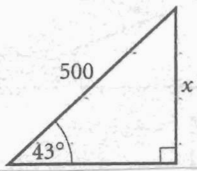
(v)


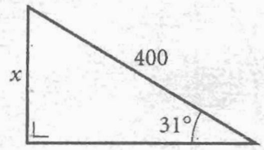
(vi)


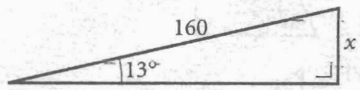
20) Find the value of x to the nearest unit:

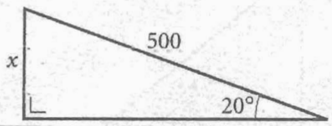
(i)


(ii)


(iii)


(iv)


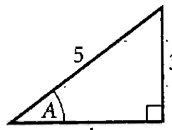
(v)


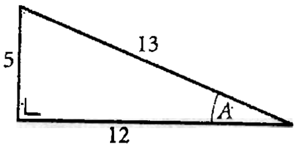
(vi)


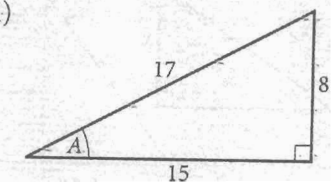
21) Find the values of these, correct to four decimal places:

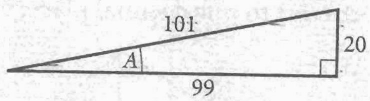
(i) tan 33° (ii) cos 23°
(iii) tan 10° (iv) cos 81°
(v) tan 66° (vi) cos 61°
(vii) tan 72° (viii) cos 11°
(ix) tan 40° (x) cos 60°

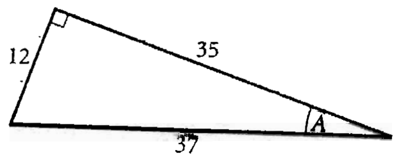
22) Write down sin A, cos A and tan A in each case:

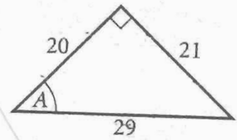
(i)


(ii)


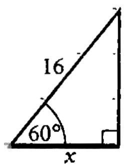
(iii)


(iv)


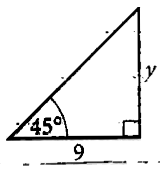
(v)


(vi)


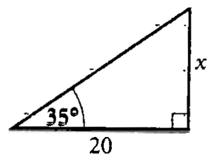
23) a) (i) Find cos 60°.
(ii) Deduce the value of x:



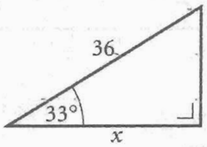
b) (i) Find tan 45°
(ii) Deduce the value of y:

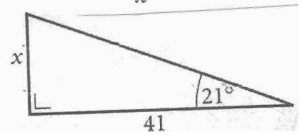


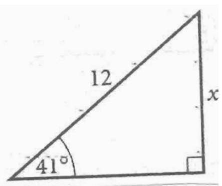
c) (i) Find tan 35° correct to one decimal place.
(ii) Deduce the value of x, to the nearest whole number:

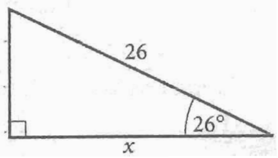


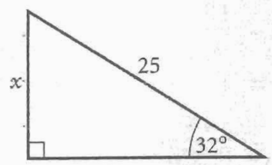
24) Find the value of x in each case correct to *one* decimal place:

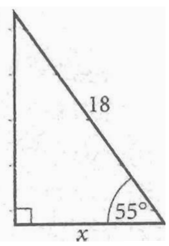
(i)


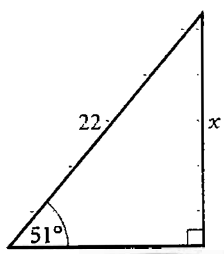
(ii)


(iii)


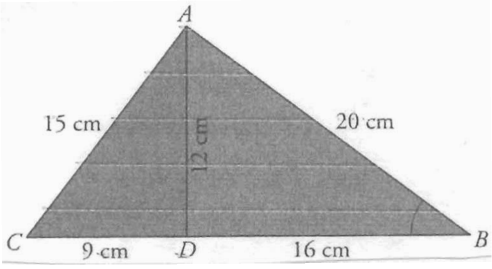
(iv)


(v)


(vii)


(viii)


25) a) Prove that $\widehat{ABC}$ and $\widehat{ADB}$ are right-angled triangles by the converse of Pythagoras' theorem
b) Calculate $\sin \hat{B}$ in both triangles and check that you get the same number



26) Fill the table:

$\sin \alpha$	0,92				0,2	
$\cos \alpha$			0,12			1/2
$tg\ \alpha$		0,75		$\sqrt{5}/2$		

27) Fill the table with the exact values for every trigonometric ratio (leave them in surd form) and for every angle ($\alpha<90°$)

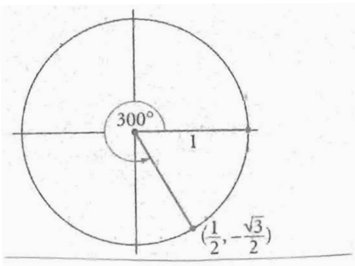
$sen\ \alpha$	1/3		
$\cos \alpha$		$\sqrt{2}/3$	
$tg\ \alpha$			2
α			

28) Prove the following (using other trigonometric identities):

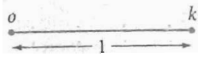
a) $$\frac{(\sin \alpha)^3 + \sin \alpha \cdot (\cos \alpha)^2}{\sin \alpha} = 1$$

b) $$\frac{(\sin \alpha)^3 + \sin \alpha \cdot (\cos \alpha)^2}{\cos \alpha} = tg\ \alpha$$

29) Use the diagram to write down the values of:
(i) cos 300°
(ii) sin 300°
(iii) tan 300°

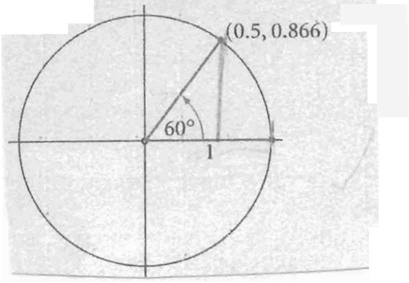


30) The diagram shows a line segment $[ok]$ such that $|ok| = 1$ unit.

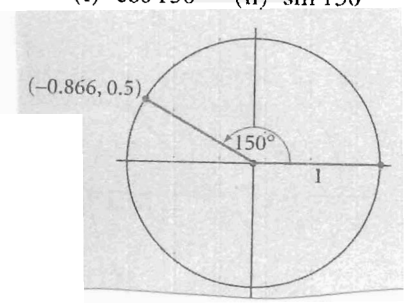


Use your protractor to find a point p on $[ok]$ such that $|op| = \cos 55°$.

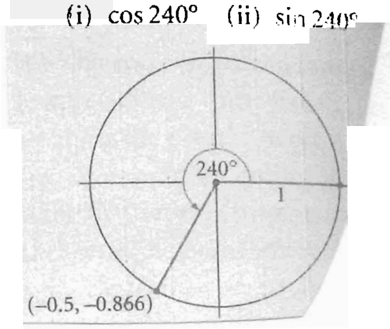
31) Use the diagram to write down the values of:
(i) $\cos 60°$ (ii) $\sin 60°$



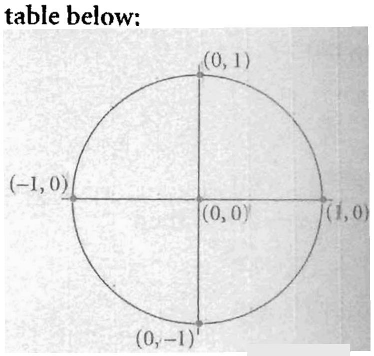
32) (a) Use the diagram to estimate the values of:
(i) $\cos 150°$ (ii) $\sin 150°$



(b) Use the diagram to estimate the values of:
(i) $\cos 240°$ (ii) $\sin 240°$



33) Use the diagram to complete the table below:



A	0°	90°	180°	270°	360°
$\cos A$					
$\sin A$					
$\tan A$		undefined		undefined	

34) (i) Write down two values of A (where $0° \le A \le 360°$) such that $\cos A = 0$.
(ii) Write down one value of A (where $0° \le A \le 360°$) such that $\sin A = -1$.
(iii) Write down one value of A (where $0° \le A \le 360°$) such that $\cos A = -1$.

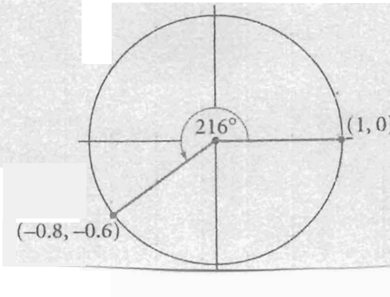
[redacted]rite down three values of A (where $0° \le A \le 360°$) such that $\sin A = 0$.
(v) Write down two values of A (where $0° \le A \le 360°$) such that $\cos A = 1$.
(vi) If $\tan \theta = 0$, find three values of θ in the range $0° \le \theta \le 360°$.
(vii) Solve for B (where $0° \le B \le 360°$): $\sin B - 1 = 0$.

35) Use the diagram to write down approximations for:

(i) cos 216°

(ii) sin 216°

(iii) tan 216°



36) Find the missing angles (to the nearest degree) in each case:

(i) $\sin A = 0.8192$

(ii) $\cos B = 0.8571$

(iii) $\tan C = 6.314$

(iv) $\sin D = 0.6947$

(v) $\cos E = 0.9816$

(vi) $\tan F = 0.4245$

(vii) $\sin G = 0.9782$

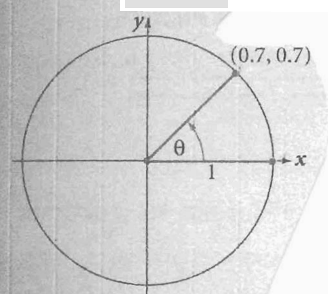
(viii) $\cos H = 0.9455$

(ix) $\tan I = 1.6$

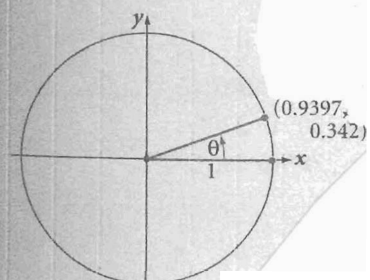
(x) $\sin J = ½$

37) [redacted] your calculator to find the value of θ (to the nearest degree) in each case:

(i)



(ii)



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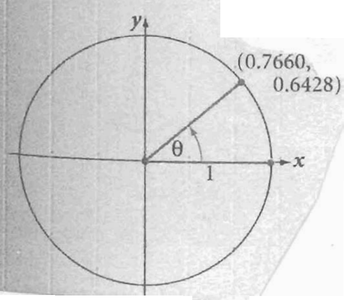

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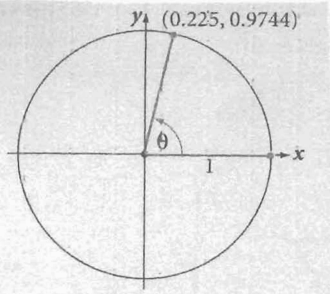
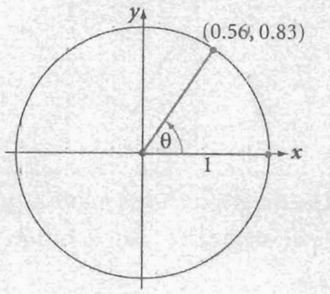
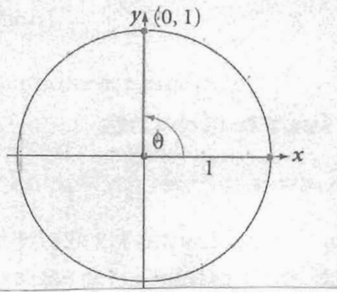


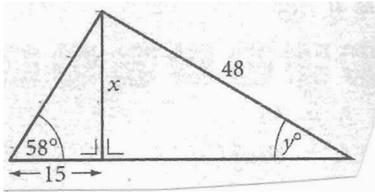
(v)



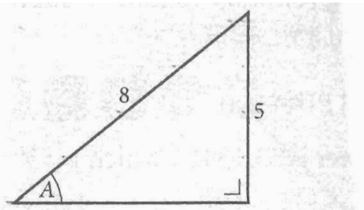
(vi)

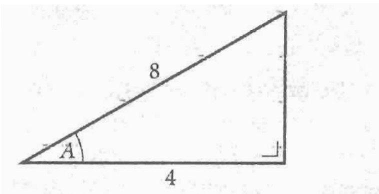


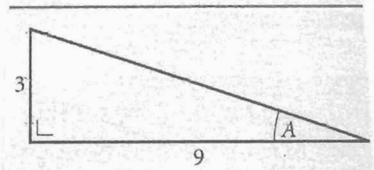
38) Find x and y to the nearest whole number.

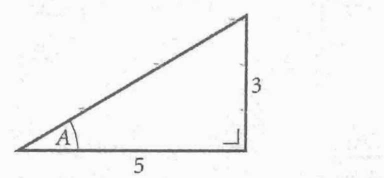


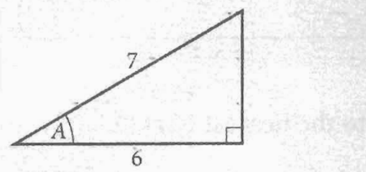
39) Find A to the nearest degree:

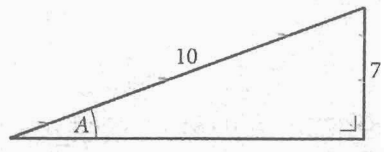


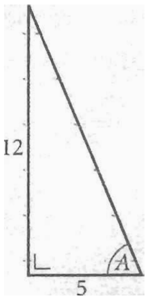


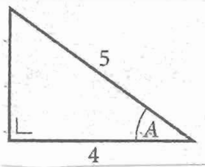


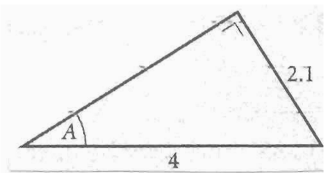












40) Solve for x $(0° \le x \le 360°)$:

a) $\sin^2 x - \sin x = 0$

b) $2\cos^2 x - \sqrt{3}\cos x = 0$

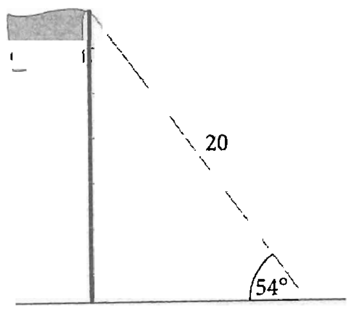
c) $3\tan x + 3 = 0$

d) $4\sin^2 x - 1 = 0$

e) $2\cos^2 x - \cos x - 1 = 0$

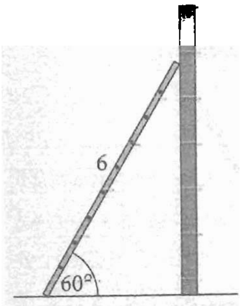
f) $2\cos^2 x - \sin^2 x + 1 = 0$

41) A vertical flagpole is fixed to the ground by a rope which is 20 metres long and which runs from the ground to the top of the pole. The rope makes an angle of 54° with the horizontal ground.



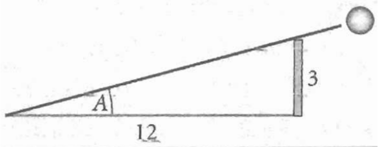
Find the height of the pole, to the nearest cm.

42) A ladder is 6 m long. It rests on horizontal ground against a vertical wall, making an angle of 60° with the ground.



How far is the foot of the ladder from the foot of the wall?

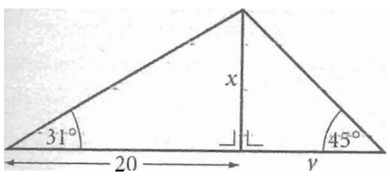
43) A vertical wall is 3 metres high. It casts a horizontal shadow 12 metres long. Find the angle of elevation A of the sun, to the nearest degree.



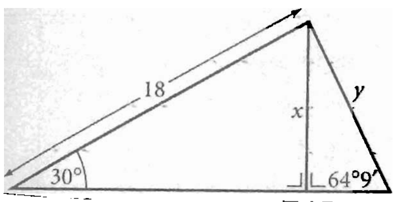
44) Find out the angles of a rhombus in which the diagonals are 12 and 8 cm long. How long is the side of the rhombus?

45) Solve for x and y to the nearest unit: You may take sines, cosines and tans correct to one decimal place (e.g. take $\tan 31° = 0.6$).

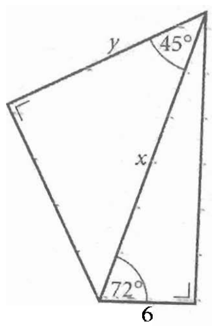
(i)



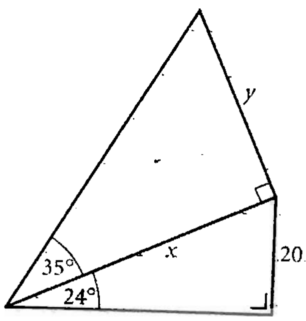
(ii)



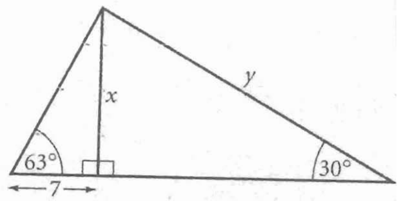
(iii)



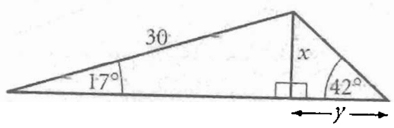
(iv)



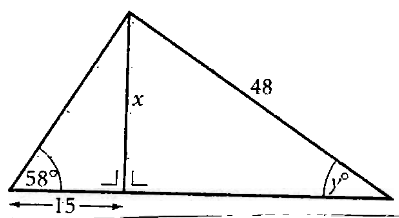
(v)



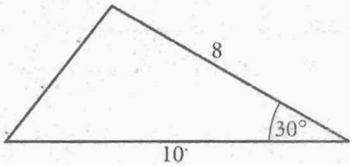
(vi)

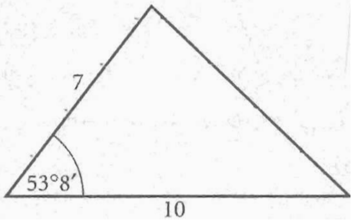


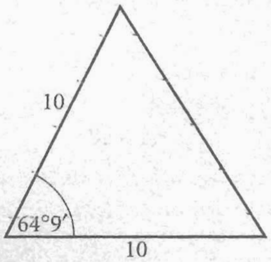
(vii)

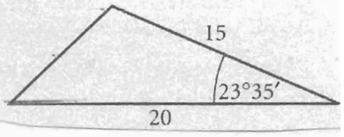


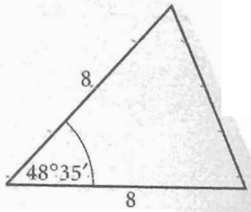
46) **Find the areas of these triangles to the nearest square unit:**

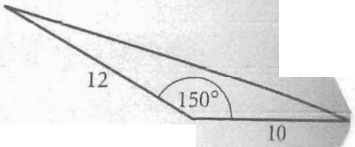
(i)


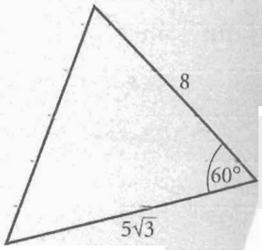
(ii)


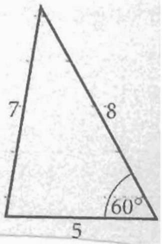
(iii)


(iv)


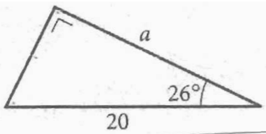
(v)


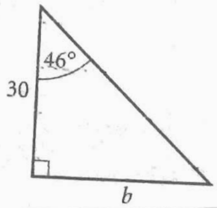
(vi)


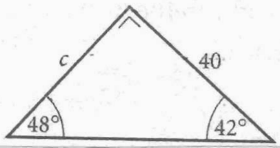
(vii)


(viii)


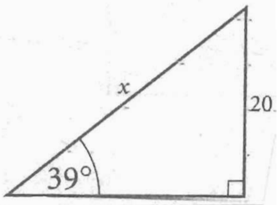
47) **Find the values of *a*, *b* and *c* to the nearest whole number:**

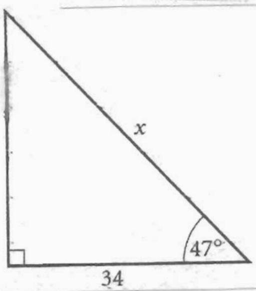
(i)


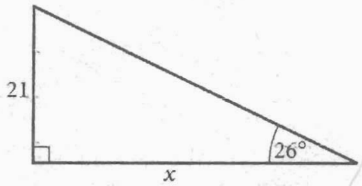
(ii)


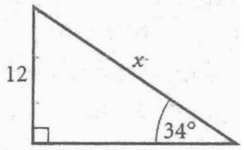
(iii)


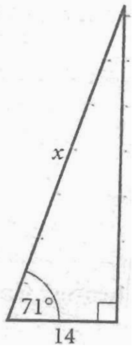
48) **Find the value of *x*, correct to two decimal places in each case:**

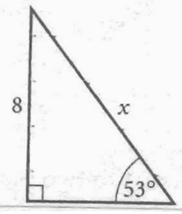
(i)


(ii)


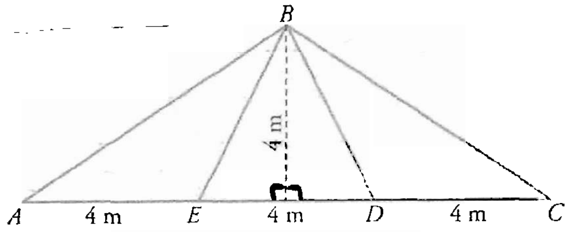
(iii)


(iv)


(v)


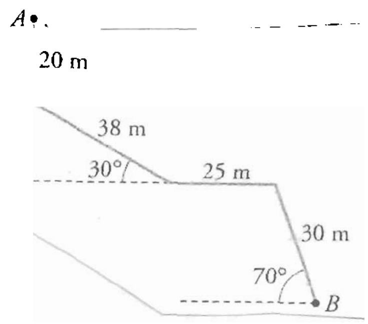
(vi)


49) A metalic structure is described below:



Calculate the lengths $\overline{AB}$, $\overline{BE}$ and find out the angles $\hat{A}$, $\hat{C}$, $\widehat{EBD}$ and $\widehat{ABC}$

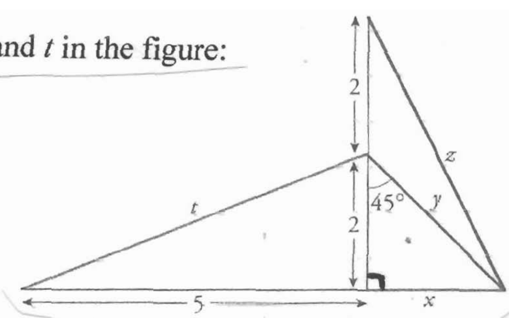
50) The speleologists use some string to calculate the depth of a cave. They stretch it and measure the length and the angle with the ground. Calculate the depth of point B.



51) A road sign tell you that the slope is 12%. Which is the angle of the road with the horizontal line? After moving 7 km along that road how many metres have we descended?

52) A hiking trail sign states that altitude is 785m. Three kilometres further the altitude is 1065m. Calculate the average slope of the trail and the angle with the horizontal line.

53) Find x, y, z, and t in the figure:



54) Solve triangle ABC in each case:

a)	b)	c)	d)	e)
$a = 27m$	$a = 8m$	$a = 10.7m$	$a = 6m$	$a = 15.3m$
$\hat{A} = 40°$	$\hat{A} = 15°$	$b = 7.5m$	$\hat{A} = 45°$	$b = 10.5m$
$\hat{B} = 73°$	$\hat{C} = 45°$	$c = 9.2m$	$\hat{B} = 30°$	$\hat{C} = 65°$

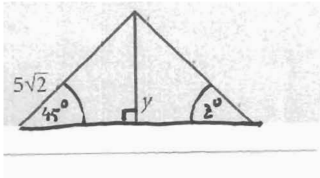
55) Give an example of an angle with:

a) positive sine and negative tangent
b) positive cosine and negative sine
c) negative tangent and negative cosine
d) positive tangent and positive sine

56) (a) Copy and complete the table (in surd form):

$\cos 45°$	$\sin 45°$	$\tan 45°$

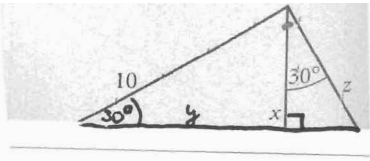
Find x, y and z:



(b) Copy and complete the table (in surd form):

$\cos 30°$	$\sin 30°$	$\tan 30°$

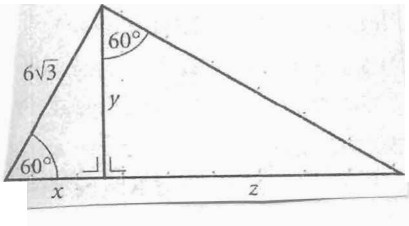
Find x, y and z in surd form:



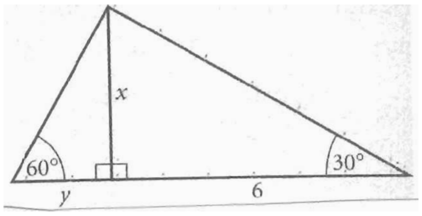
(c) Copy and complete the table (in surd form):

$\cos 60°$	$\sin 60°$	$\tan 60°$

Find x, y and z in surd form:



57) Find the value of x and the value of y (in surd form):



58) a) If $A = 45°$, evaluate $\sin^2 A + \cos^2 A$

b) Evaluate $\sin^2 60° + \cos^2 60° + \tan^2 60°$.

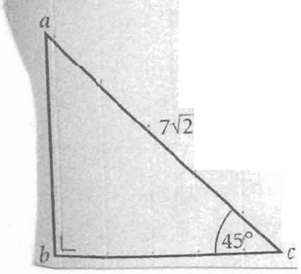
c) Evaluate $\tan^2 30° + \tan^2 45° + \tan^2 60°$.

59) Find:

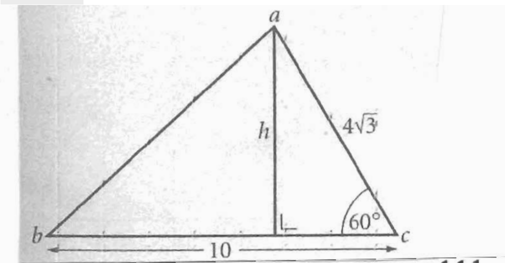
(i) $|ab|$.

(ii) $|bc|$.

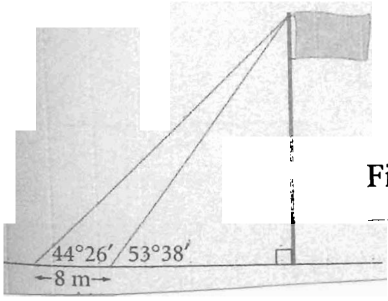
(iii) area $\triangle abc$.



60) Find h, the perpendicular height of $\triangle abc$. Hence find the area of $\triangle abc$.



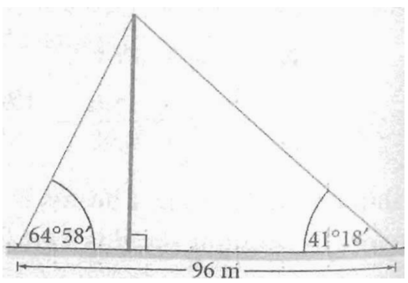
61) A vertical flagpole stands on horizontal ground. It is kept upright by two wires, as shown.



Find the height of the pole (to the nearest metre).

62) If $\cos A = -\frac{1}{\sqrt{2}}$, find two values of A where $0° < A < 360°$.

63) A vertical pole stands on horizontal ground. It is kept in position by two ropes: one long and one short. The ropes make angles of 64°58′ and 41°18′ with the ground. The ropes are tied at points on the ground, 96 m apart.

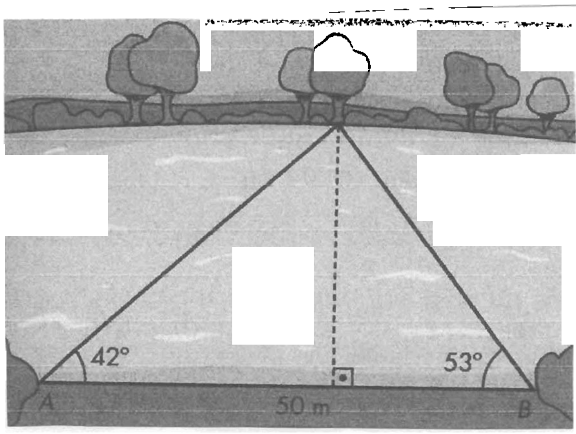


Find (to the nearest metre):

(i) the length of the shorter rope.

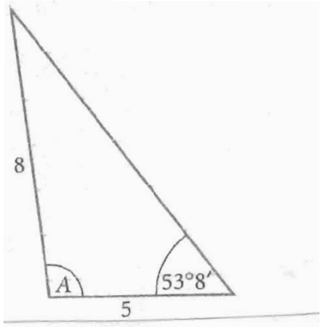
(ii) the height of the pole.

64) Look at the data John collected in order to calculate the river's width.

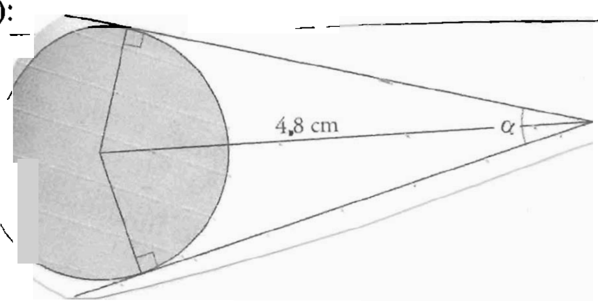


LESSON 4

65) Find angle A and hence find the area of this triangle correct to two decimal places.



66) The diameter of a two-euro coin is 2.5 cm.
Find out the angle between the tangent lines that cross at a point 4.8 cm far from the centre (as shown below):



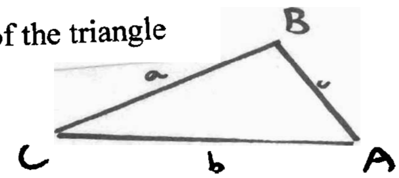
67) Data:

$c = 30cm$

$\hat{A} = 40°$

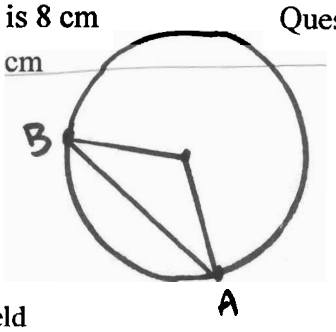
$\hat{B} = 105°$

Question: the area of the triangle



68) Data: the radius of the circle is 8 cm
the segment $\overline{AB}$ is 10 cm

Question: the angle $\widehat{AOB}$



69) A farmer owns a triangular field $\triangle pqr$, as shown. She wants to sow 1140 m^2 of barley in triangular piece qrx, and vegetables in the rest of the field.

Calculate the required distance $|qx|$.

Find the length $|pr|$ and hence find the area which will be sown with vegetables (to the nearest square metre).