NUMBER SETS			INTERNAL OPERATIONS
NAME	SYMBOL	EXAMPLES	
Natural numbers	N	1, 2, 3, 4, 5, 6, 7, 2145, 10000…	+ ·
Integer numbers	Z	…,– 100, – 5,– 2, – 1, 0, 3, 7,…	+ · –
Rational numbers	Q	$\ldots -20, -\frac{9}{5}, -1, 0, 2, \frac{3}{7}, 4, \ldots$	+ · – :
Real numbers	$\Re$	$\ldots, -\frac{9}{5}, -\pi, -6, 0, 2, \frac{3}{7}, \sqrt{3}, \ldots$	+ · – : $\sqrt{\ }$ ↑

Note: A real number is called "irrational" if it is not a rational number.
Note: in English, "whole numbers" means the set consisting of the natural numbers and the zero.

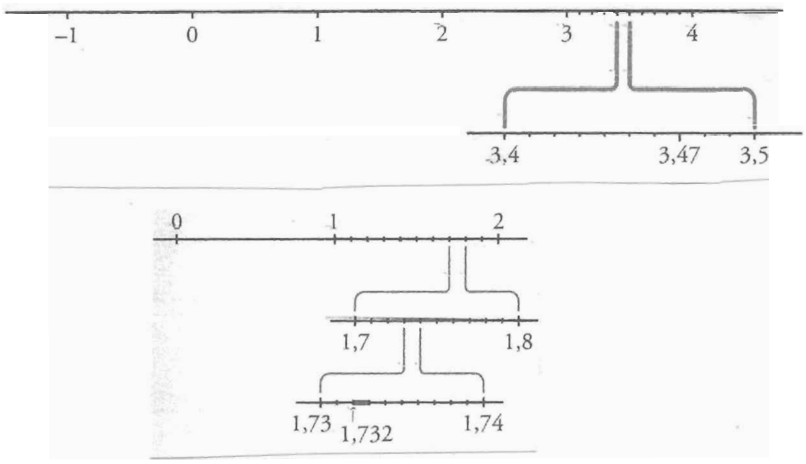
Decimal numbers are the same as real numbers:

- A decimal number with no digits on the right of the decimal point is an integer.
- A terminating decimal number or a repeating decimal number is a rational.
- A non-terminating, non-repeating decimal number is not a rational; it is called an irrational number.

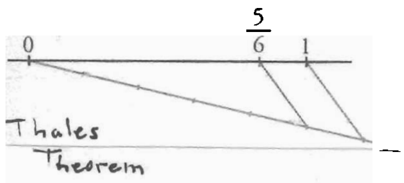
Note: In England, the symbol used to separate the integer part of a decimal number from its fractional part is a decimal point.

Location on the real line:

- Integers are placed leaving always the same distance between each number and the following. Zero is in the middle, positives are on the right and negatives are on the left. The numbers grow from left to right.
- A decimal number is pictured by repeatedly fractioning each unit into ten smaller-order units.



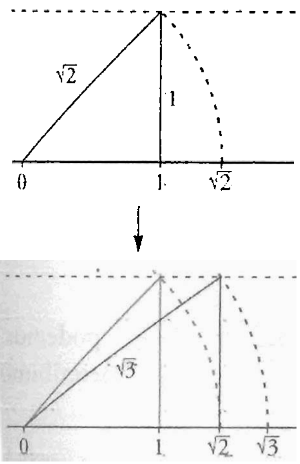
- Proper fractions are pictured dividing the unit into as many parts as the denominator tells and moving to the right (to the left, if negative) as many parts as the numerator tells.



- Improper fractions are first changed into mixed numbers. Then the fractional part is pictured on the unit segment following the integer.

- Exact method to locate numbers like $\sqrt{n}$,$(n \in Z)$ on the real line: consider $n = \left(\sqrt{n-1}\right)^2 + 1^2$ so we can write $\sqrt{n} = \sqrt{\left(\sqrt{n-1}\right)^2 + 1^2}$; apply Pythagoras Theorem to a right-angled triangle with short sides $\sqrt{n-1}$ and 1; the hypothenuse should be $\sqrt{n}$; use a pair of compasses to bring that measure down onto the real line.

$\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{2+1} = \sqrt{3}$



Intervals of real numbers

We need to be able to talk easily about certain subsets of $\Re$.

We say that $I \subset \Re$ is an **open interval** if $I = (a,b) = \{x \in \Re \mid a < x < b\}$ ("the numbers between a and b")
We say that $I \subset \Re$ is a **closed interval** if $I = [a,b] = \{x \in \Re \mid a \le x \le b\}$ ("the previous plus a and b")
We call **half-open intervals** to sets like these: $(a,b] = \{x \in \Re \mid a < x \le b\}$ and $[a,b) = \{x \in \Re \mid a \le x < b\}$

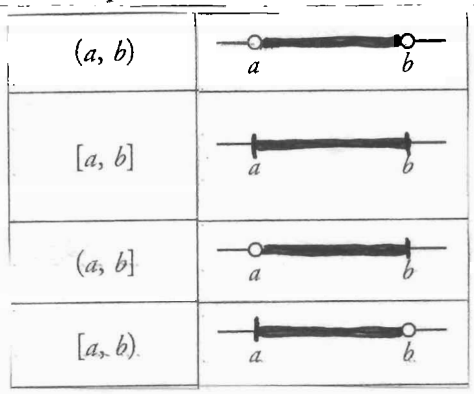
Note: an open interval excludes its end points, but contains all the points in between; in contrast a closed interval contains both its end points. It is trivial that $[a,b] = (a,b) \cup \{a\} \cup \{b\}$

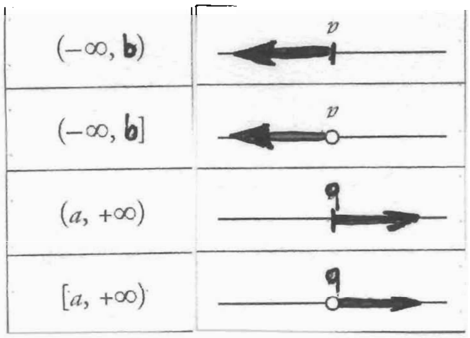
The two end points a and b are points in $\Re$. It is sometimes convenient to allow also the possibility $a = -\infty$ and $b = +\infty$ to express **infinite intervals** (also called **unbounded intervals**):

$(-\infty, b) = \{x \in \Re \mid x < b\}$	$(-\infty, b] = \{x \in \Re \mid x \le b\}$	$(a, +\infty) = \{x \in \Re \mid a < x\}$	$[a, -\infty) = \{x \in \Re \mid a \le x\}$
"numbers smaller than b"	"b and the smaller than b"	"numbers greater than a"	a and the greater than a

Note: It is trivial that $\Re = (-\infty, +\infty)$

We can represent intervals on the real line:





A fraction may be changed into a decimal number

When we divide numerator by denominator we can obtain:

- An integer number
- A terminating decimal number
- A repeating non-terminating decimal number (recurring decimal)

Note: Suppose the fraction is irreducible. If denominator is factorized in prime factors and these prime factors are only numbers two and/or five, the outcome will be a whole number or a terminating decimal number.
Note: Suppose the fraction is irreducible. If denominator is factorized in prime factors and these prime factors are numbers two and/or five as well as any other numbers, the outcome will be a mixed recursive decimal number.
Note: Suppose the fraction is irreducible. If denominator is factorized in prime factors and these prime factors are numbers different from two or five, the outcome will be a repeating non-terminating decimal number with all the decimal part digits being recursive.

Some decimal numbers may be changed into fractions (rational numbers)

All rational numbers give decimal parts that either terminate or recur.

-A whole number
Write down the number divided by 1

-A terminating decimal number
Remove the decimal point and use the decimal number as the numerator.
The denominator is the number 1 followed by as many zeros as decimal places in the number.
Reduce the fraction.

-A repeating non-terminating decimal number (all the digits in the decimal part are recursive).
Count the number of digits that are repeating and write this power of ten
Multiply the decimal number by it and subtract the original number: it is the numerator
Denominator = the power of 10 minus 1

-A repeating non-terminating decimal number (there are no-repeating digits in the decimal part)
Count the number of decimal digits and write this power of ten
Count the number of digits that are not repeating and write this power of ten
Multiply the decimal number by both powers and subtract: it is the numerator
Denominator = the difference between both powers of 10

Some decimal numbers can not be changed into fractions (irrational numbers)

The numbers whose decimal part continues forever with no pattern are irrational. They can not be changed into fractions.

- Some irrational numbers are trascendental numbers like π
- The surds that are not terminating numbers are irrational $\sqrt{5}$, $\sqrt[3]{6}$, $\sqrt{\frac{2}{3}}$

EXERCISES

1) Write a rational number and an irrational number between M and N in each case:

a) $M = \frac{1}{2}$ and $N = \frac{1}{3}$

b) M = 0.438 and N = 0.439

c) $M = 0,\overline{31}$ and $N = 0,\overline{32}$

2) a) Locate the following numbers exactly on the real line: –2; 3,75; $\sqrt{5}$; 0,666...

b) Locate the number Φ = 1.618... approximately on the real line.

3) a) Write as an interval the set of numbers between 3 and 10 and represent it on the real line.

b) Write as an interval the set of numbers between 1 and 7, both of them included, and represent it on the real line.

c) Write as an interval the set of numbers between 2 and 21, number 21 included, and represent it on the real line.

d) Write as an interval the set of numbers between –2 and 0, number –2 included and represent it on the real line.

e) Represent the set [3,10] on the real line and define it (tell which real numbers are in it)

f) Represent the set (1,7] on the real line and define it (tell which real numbers are in it)

g) Represent the set (3,10] on the real line and define it (tell which real numbers are in it)

h) Represent the set (2,21) on the real line and define it (tell which real numbers are in it)

k) Represent the set [3,10) on the real line and define it (tell which real numbers are in it)

l) Represent the set [–2 ,0] on the real line and define it (tell which real numbers are in it)

m) Represent the set (1,7) on the real line and define it (tell which real numbers are in it)

n) Represent the set [1,7) on the real line and define it (tell which real numbers are in it)

o) Represent the set [2,21) on the real line and define it (tell which real numbers are in it)

p) Represent the set [2,21] on the real line and define it (tell which real numbers are in it)

q) Represent the set (–2 ,0) on the real line and define it (tell which real numbers are in it)

r) Represent the set (–2 ,0] on the real line and define it (tell which real numbers are in it)

4) Change the following decimal numbers into fractions (cancelling as much as possible):

0.0625 10.75 1.024 0.888... 1.3535... 0.02525...

5) Order and locate on the real line the following numbers:

-2.5 $\quad 0.55$ $\quad \frac{-7}{3}$ $\quad \frac{15}{6}$ $\quad 3.1$ $\quad 0.5$ $\quad 0.49$

6) Calculate and simplify: $\frac{4}{10}+\frac{3}{100}-\frac{5}{1000}+\frac{6}{10000}-\frac{345}{1000000}=$

7) Calculate and simplify:

a) $\dfrac{(0'45-1)^2-\left(0'333...-\frac{2}{3}\right)^3}{3\cdot 10^2}=$

b) $\left(0'3232...-\frac{1}{3}\right)\cdot\left(\frac{2}{3}-4'5\right)+0'0455...(1-0'45)\cdot\frac{3}{5}=$

8) Which of these are irrational numbers? {2½, 0.9, –3, π, $\sqrt{11}$}

Exponential notation.

- ~~Exponential notation is a method of writing numbers that have many zeros.~~
- The basis of exponential notation is the power of ten.
- A number written in exponential notation has two parts.

  The first part is a real number

  The second part is a power of ten.

  Examples: $15{,}600{,}000 = 156 \times 10^5$ or else 156E5

  $0{,}0000000072 = 72 \times 10^{-10}$ or else 72E-10

Scientific notation.

- ~~Scientific notation is a method of writing very large and very small numbers.~~
- ~~The basis of scientific notation is the power of ten.~~
- A number written in scientific notation has two parts.

  The first part is a number between 1 and 10 called "mantissa".

  The second part is a power of ten.

  Examples: $15{,}653 = 1.5653 \times 10^4$ or else 1.5653E4

  $0{,}0000000072 = 7.2 \times 10^{-9}$ or else 7.2E-9

$$\boxed{a \times 10^n \text{ where } 1 \le a < 10 \text{ and } n \in Z.}$$

- Converting numbers into scientific notation

  First move the decimal place so that only one integer is on the left side of the decimal.

  Then, following the number, write a multiplication sign.

  ~~Then, the number 10 raised to the number of places you moved the decimal comma.~~

  If the number is large, the exponent is positive, if the number is small the exponent is negative.

Calculators can deal with scientific notation by putting the calculator into *scientific mode* or by using the button. (Or [EE] button)

To get 3.1×10^4, press 

To get 5.38×10^{-2}, press 

Note: 6 · 4 Exp 8

This appears as 6.4^{08} on your screen. You have to understand that this is the calculator's way of writing 6.4×10^8. (It does **not** mean 6.4^8!)

EXERCISES

9) a) The mass of the electron is 0.000 000 000 000 000 000 000 000 000 91 Kg. approximately. Write this quantity in exponential notation.

b) Andromeda constelation is two million light-year far from Earth. Calculate that distance in Km and write it in scientific notation.

c) You have more than fourteen thousand million of neurons in your brain. Write this number in scientific notation.

10) **Here are six numbers from science. Write them in scientific notation:**

(i) The number of years since the extinction of dinosaurs: 64 000 000
(ii) The speed of a dropped stone after three seconds (in metres per second): 29.43
(iii) The melting point of diamond (in degrees Celsius): 3500
(iv) The speed of sound in km/h: 1190
(v) The speed of light (in m/s): 300 000 000
(vi) The diameter of the Earth (in metres): 12 756 000

11) **Write these in the form $a \times 10^n$ where $1 \le a < 10$ and $n \in Z$:**

(i) 2600 (ii) 0.034
(iii) 51 000 (iv) 0.005
(v) 610 (vi) 0.0002
(vii) 0.523 (viii) 520 000
(ix) 0.000 000 046
(x) 23 000 000

12) **Write each of these as a decimal number:**

(i) 7.1×10^3 (ii) 8.14×10^5
(iii) 9.7×10^{-3} (iv) 1.7×10^{-2}
(v) 2.85×10^{-1} (vi) 7.74×10^3
(vii) 6×10^{-5} (viii) 1×10^{-4}
(ix) 4×10^{-1} (x) 2.5×10^4

13) **Evaluate each of these and give the answer in scientific notation:**

(i) $4.1 \times 10^3 + 3.5 \times 10^2$
(ii) $1.9 \times 10^4 - 4.5 \times 10^3$
(iii) $(2 \times 10^3) \times (3 \times 10^4)$
(iv) $\dfrac{5.1 \times 10^8}{3 \times 10^3}$
(v) $5.6 \times 10^2 + 4.8 \times 10^2$
(vi) $3.8 \times 10^4 - 5 \times 10^3$
(vii) $2.3 \times 10^2 - 9 \times 10^1$
(viii) $6 \times 10^3 - 6 \times 10^2$
(ix) $5.4 \times 10^2 + 4.5 \times 10^4$
(x) $1 \times 10^5 - 2 \times 10^4$
(xi) $(2 \times 10^2) \times (3.1 \times 10^3)$
(xii) $(1.5 \times 10^4) \times (4 \times 10^2)$
(xiii) $(2.5 . 10^2) . (3 . 10^7)$
(xiv) $(3 . 10^2) \times (4 . 10^5)$
(xv) $(5 . 10^2) \times (3.2 . 10^6)$
(xvi) $(2.1 . 10^5) . (6.1 . 10^8)$

14) Evaluate the following and give your answer in scientific notation:

(i) $\dfrac{8\times10^5}{2\times10^2}$

(ii) $\dfrac{6.3\times10^7}{3\times10^3}$

(iii) $\dfrac{8.5\times10^5}{5\times10^1}$

(iv) $\dfrac{4\times10^6}{8\times10^2}$

(v) $\dfrac{5.1\times10^{12}}{1.7\times10^5}$

(vi) $\dfrac{9.8\,.\,10^{12}}{7\,.\,10^3}$

15) Evaluate these in scientific notation:

(i) $6.4\times10^1+9.8\times10^0$

(ii) $5\times10^{-2}-8\times10^{-4}$

(iii) $(2.1\times10^{-3})^2$

(iv) $\dfrac{3\times10^5}{5\times10^{-3}}$

16) Write these in the form $a\times10^n$, where $1\le a<10$ and $n\in N$:

(i) $\dfrac{(2.1\times10^2)\times(4\times10^7)}{1.4\times10^4}$

(ii) $\dfrac{(1.6\times10^5)\,.\,(9\times10^2)}{(1.2\times10^2)\,.\,(4\times10^1)}$

17) Find the value of n if

$\dfrac{66}{0.011}=6\times10^n$.

b) Express $\dfrac{(2\times10^3)^2}{(5\times10^{-4})}$ in the form 8×10^n, where $n\in Z$.

c) Find the value of n if

$\dfrac{0.056}{80}=7\times10^n$, where $n\in Z$.

d) Express $\dfrac{(4\times10^{-2})^3}{(8\times10^{-4})}$ as $a\times10^n$ where $1\le a<10$ and $n\in Z$.

e) Evaluate $\dfrac{200}{0.16}$ and write your answer in scientific notation.

f) Evaluate $\dfrac{250}{0.005}$ and write your answer in scientific notation.

g) Add $14.38275+10.61725$ and write your answer in the form $a\,.\,10^n$ where $1\le a<10$, $n\in N$.

Accuracy

When expressing a number with respect to a fixed number of decimal places we are really deciding on the accuracy to which we want to work. One often uses the shorthand '**expressed to nD**' to emphasise that a number has been **expressed to n decimal places.**

Examples:	expressed to 5 decimal places (expressed to 5D)	12.50913	94,267.60992	0.00043
	expressed to 3 decimal places (expressed to 3D):	3.142	2.718	0.577

Significant figures

Reading from the left the first non zero digit is the most significant and will be called the first significant digit. The digit to the right of the first significant digit is the second significant digit and so on.

If we **represent a number to n significant digits** we say it is '**displayed to nS**'

Examples:	expressed to 5 significant digits (expressed to 5S)	123.45	0.000032004	12.002
	expressed to 3 significant digits (expressed to 3S)	0.000305	123000	1.06

Approximation

Often we wish to rewrite a number with less significant digits. There are two methods to do this.

- **Chopping** (one simply discards the superfluous digits)

Examples: $163.190385 \rightarrow 163.1903$ $\quad -34.978992 \rightarrow -34.9789$ $\quad 0.000299 \rightarrow 0.0002$

Rounding (we first chop the number at nD; if the digit following the last significant digit was 5 or higher, we increase the last significant digit in one unit)

Examples: $163.190385 \rightarrow 163.1904$ $\quad -34.978992 \rightarrow -34.9790$ $\quad 0.000299 \rightarrow 0.0003$

$26.288359 \rightarrow 26.2884$ $\quad 26.288459 \rightarrow 26.2885$

Error

When one has to deal with numbers which have been rounded, we keep note of the error involved.

Actual absolute error

Given some number X and its approximation x the actual **absolute error** is: $\varepsilon = |X - x|$

The maximal absolute error for a number given to n decimal places (to nD) is 0.5×10^{-n}

It is the **maximal error bound.**

Example: If we use 3.14159 as a decimal approximation of π, the actual absolute error is $|\pi - 3.14159|$. It is less than 0.000005 so this is the maximal error bound.

Actual relative error

Given some number X and its approximation x the actual **relative error** is: $r = \left|\frac{\varepsilon}{X}\right| = \left|\frac{X-x}{X}\right|$

The maximal relative error for a number given to n decimal places (to nD) is: $\dfrac{0.5 \times 10^{-n}}{|x|}$

It is the **maximal relative error bound**

Examples:

1) An absolute error of 0.5 is not very important if the number is X = 62,500
But it is very high if the number is X = –2
The relatives errors would be 0.000008 in the first approximation and 0.25 in the second.

2) If we use 3.14159 as a decimal approximation of π
The relative error would be less than $\dfrac{0.000005}{3.14159}$, so less than $1.592 \times 10^{-6} = 0.000001592$

EXERCISES

18) a) Express to a sensible number of significant digits:

The number of visitors to an arts exhibition is	1,345,589	people.
The number of participants in a demostration against pollution is	1,345,589	people.
The number of bacteria living in 1 dm3 of culture liquid is	203,305,123	bacteria
The number of water drops that are there in a water pool is	8,249,327,741	drops
The number of grains that are there in a sack of sand is	2,937,248	grains

b) Calculate the maximal error bound and the maximal relative error bound of the previous approximations

19)

a) If €1 is worth $1.132, find the value of $1 correct to the nearest euro cent.

b) If €1 is worth 8.3624 Hong Kong dollars, find the value of 1 Hong Kong dollar, to the nearest euro-cent.

c) If 1 Canadian dollar is worth €0.6173, find the value of €1 in Canadian dollars (correct to two decimal places).

d) If a car uses up 0.077 litres of petrol in travelling 1 km, how many kilometres per litre does this car travel (to the nearest kilometre)?

e) If 1 pound = 2.2 kilograms what is 1 kilogram equal to in pounds? Give your answer correct to three decimal places.

20)

a) Find, correct to two decimal places $\frac{1}{5.885}$

b) Find, correct to three decimal places $\frac{1}{4.329}$

c) Find, correct to three decimal places $\frac{1}{3.5 + 3.034}$

d) Evaluate $\frac{1}{3.3^3}$ correct to four decimal places.

e) Calculate, correct to two decimal places $\frac{1}{5 - 0.833}$

f) Evaluate $\frac{1}{\sqrt{2.56}}$

21) Copy and complete the following table, giving the true answer correct to four decimal places.

Question	Approximation	Approximate answer	True answer
(i) $\frac{1}{4.848}$	$\frac{1}{5}$	0.2	
(ii) $\frac{1}{2.136}$	$\frac{1}{2}$		
(iii) $\frac{1}{4.232}$			
(iv) $\frac{1}{8.039}$			
(v) $\frac{1}{9.888}$			

22) Evaluate $850\left(1+\frac{1}{11}\right)^4$, rounding your answer off to the nearest hundred.

23) a) If $p = 4.375$, evaluate $\frac{1}{\sqrt{p}+p}$ correct to 4 decimal places.

b) If $x = 7$ and $y = 24$, evaluate $\frac{1}{\sqrt{x^2+y^2}}$

c) Evaluate correct to one decimal place $\sqrt{x^2+\frac{1}{x}}$ if $x = 0.7412$.

d) Evaluate (correct to two decimal places) $\frac{1}{x^2-\sqrt{x}}$ if $x = 2.74$.

e) If $y = 2.071$, evaluate $2y^3 + 5y^{1/2}$ correct to two decimal places.

f) If $x = 8.32$, evaluate (correct to the nearest whole number) $\frac{1}{x}+\sqrt{x}$

g) If $y = 7.472$, evaluate $\frac{1}{y^2+10}$ to three decimal places.

h) Evaluate $\frac{1}{p}+p^2+\sqrt{p}$ correct to one decimal place, if $p = 2.532$.

i) Evaluate $\frac{1}{x}+x^2+\sqrt{x}$ correct to two decimal places, if $x = 4.855$.

j) If $y = 3.692$ evaluate $\frac{1}{y^2+\sqrt{y}}$ to four decimal places.

k) If $x = 44.72$, find $\sqrt{\frac{1}{x}-\frac{1}{x^2}}$ correct to four decimal places.

l) Evaluate $\sqrt{\frac{1}{x}+\sqrt{y}+z^2}$ (correct to two decimal places) if $x = 2.478\times10^{-1}$, $y = 4.41\times10^2$ and $z = 1.779$.

m) If $p = 0.8756$, evaluate $(p^{-1}+p^{1/2})$ correct to three decimal places.

24) (i) Write $\frac{1}{81.3}$ in the form $a\times10^n$, where $0 \le a < 10$ and $n \in Z$, writing a correct to two decimal places.

(ii) The mass of the Earth is 6×10^{24} kilograms, which is 81.3 times greater than the mass of the Moon.

Find the mass of the Moon in the form $a\times10^n$ where $1 \le a < 10$, $n \in Z$, and a is written correct to two decimal places.

25) a) (i) By writing each number correct to the nearest whole number, estimate the value of n where

$$n = \frac{4\sqrt{35.7}}{2.1 + 0.85}$$

showing each step.

(ii) Evaluate n correct to three decimal places, using a calculator.

b) (i) By writing each number correct to the nearest integer, estimate the value of p where

$$p = \frac{(2.9)^3 + \sqrt{25.2}}{2\sqrt{4.08}}.$$

(ii) Evaluate p, correct to three decimal places, using a calculator.

c) (i) By writing each number to the nearest whole number, estimate the value of $k > 0$, where

$$k^2 = \frac{(4.038)^3 + 4(2.912)^2}{2\sqrt{3.877}}$$

(ii) Evaluate k correct to three decimal places, using a calculator.

26) a) If $x^2 - \frac{1}{y} = z^2$,

(where x, y and z are positive):

(i) write x in terms of y and z.

(ii) evaluate x, correct to one decimal place, if $y = 2.222 \times 10^{-2}$ and $z = 3.333$.

b) If $x(\sqrt{y} - y^2) = 1$, evaluate x correct to two decimal places when $y = 0.4874$.

c) If $a^3 + b^2 = \sqrt{\frac{1}{c}}$, evaluate b correct to two decimal places, when $b > 0$, $a = 0.9165$ and $c = 0.5643$.

d) (i) By writing each number to the nearest whole number, estimate the value of t where

$$= \frac{90.1}{5.89} + 5.06\sqrt{49.31}.$$

(ii) Deduce an estimate for y if $2y^2 = t$ and $y > 0$.

(iii) Find the value of y, correct to four decimal places if

$$2y^2 = \frac{90.1}{5.89} + 5.06\sqrt{49.31}.$$

e) (i) If $x^4 = \sqrt{y^2 - 1500} + 5z$, express y in terms of x and z, given that they are all positive numbers.

(ii) Evaluate y correct to two significant figures if $x = 12.918$ and $z = 105.106$.

Power rules

- Exponents or 'powers' are a process of repeated multiplication.
- Exponents are normally written in the form a^b, where 'a' is the base and 'b' is the exponent.
- We pronounce a^b as 'a to the power of b' or 'a to the b' or 'a exponent b'.

When calculating with 'powers' use the following rules:

$a^1 = a$

$a^0 = 1$

Same base

$a^b \cdot a^c = a^{b+c}$

$a^b : a^c = \frac{a^b}{a^c} = a^{b-c}$

Same exponent

$p^b \cdot q^b = (p \cdot q)^b$

$p^b : q^b = \frac{p^b}{q^b} = (p:q)^b = \left(\frac{p}{q}\right)^b$

Exponent of an exponent

$(a^b)^c = a^{b \cdot c}$

Negative exponents

$a^{-b} = \frac{1}{a^b}$

$\left(\frac{p}{q}\right)^{-b} = \left(\frac{q}{p}\right)^b$

Fractional exponents

$a^{\frac{1}{c}} = \sqrt[c]{a}$ $\qquad a^{\frac{b}{c}} = \sqrt[c]{a^b}$

N^{th} root of a number

The expression $\sqrt[n]{a}$ (**"n^{th} root of a"**) means: a number such that raised to the n^{th} power equals "a"

$$\boxed{\sqrt[n]{a} = R \Leftrightarrow R^n = a}$$

"a" is called **the rooted**

"n" is called **the index** (when index is 2, it does not have to be written)

$\sqrt{\ }$ is the **radical bar**

Examples:	$\sqrt[5]{32} = 2$	because	$2^5 = 32$
	$\sqrt[4]{81} = 3$	because	$3^4 = 81$
	$\sqrt[3]{-125} = -5$	because	$(-5)^3 = -125$
	$\sqrt[3]{64} = 4$	because	$4^3 = 64$
	$\sqrt{100} = 10$	because	$10^2 = 100$

note: a 2-index root is called "**square root**"
a 3-index root is called "**cube root**"
a 4-index root is called "**fourth root**", a 5-index root is called "**fifth root**", and so on.

Surds

A **surd** is and expression involving roots.

Sometimes it is useful to work with them, rather than using an approximate decimal value. Surds can be manipulated just like algebraic expressions.

When asked to give the exact value, approximate decimal answers will not do and you will have to manipulate surds in order to give a final answer in simplified surd form.

Example:	$\sqrt[4]{3} \cdot \sqrt[4]{27} = 1.31607 \cdot 2.27951 = 2.99999$	is not exact
	$\sqrt[4]{3} \cdot \sqrt[4]{27} = \sqrt[4]{3 \cdot 27} = \sqrt[4]{81} = 3$	is exact

Surds as indices

A n^{th} root can be written as an exponent: $\boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$

The laws of indices also apply to any n^{th} root. We can use these laws to manipulate surds.

note: $\sqrt[n]{a^p} = a^{\frac{p}{n}}$ as well

1. $\sqrt[np]{a^p}=\sqrt[n]{a}$ as $\sqrt[np]{a^p}=a^{\frac{p}{np}}=a^{\frac{1}{n}}=\sqrt[n]{a}$

Uses

- Simplify radicals. For example: $\sqrt[4]{9}=\sqrt[4]{3^2}=\sqrt{3}$
- Change surds to have the same index. For example to compare $\sqrt[3]{586}$ with $\sqrt{70}$:

$\sqrt[3]{586}=\sqrt[6]{586^2}=\sqrt[6]{343396}$, $\sqrt{70}=\sqrt[6]{70^3}=\sqrt[6]{343000}$ therefore, $\sqrt[3]{586}>\sqrt{70}$

2. $\sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}$ as $\sqrt[n]{a\cdot b}=(a\cdot b)^{\frac{1}{n}}=a^{\frac{1}{n}}\cdot b^{\frac{1}{n}}=\sqrt[n]{a}\cdot\sqrt[n]{b}$

Uses

- Take a factor out of the radical. For example: $\sqrt{18}=\sqrt{9\cdot 2}=\sqrt{9}\cdot\sqrt{2}=3\sqrt{2}$
- Write two radicals under the same radical bar. For example: $\sqrt{15}\cdot\sqrt{20}=\sqrt{15\cdot 20}=\sqrt{300}$

3. $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ as $\sqrt[n]{\frac{a}{b}}=\left(\frac{a}{b}\right)^{\frac{1}{n}}=\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Uses

- Together with rules number **1** and **2** this rule allows to write surds products and quotients under the same radical bar. For example: $\frac{\sqrt{3}\cdot\sqrt[3]{4}}{\sqrt[6]{24}}=\frac{\sqrt[6]{3^3}\cdot\sqrt[6]{4^2}}{\sqrt[6]{2^3\cdot 3}}=\sqrt[6]{\frac{3^3\cdot 2^4}{2^3\cdot 3}}=\sqrt[6]{2\cdot 3^2}=\sqrt[6]{18}$

4. $\left(\sqrt[n]{a}\right)^p=\sqrt[n]{a^p}$ as $\left(\sqrt[n]{a}\right)^p=\left(a^{\frac{1}{n}}\right)^p=a^{\frac{1}{n}\cdot p}=\left(a^p\right)^{\frac{1}{n}}=\sqrt[n]{a^p}$

5. $\sqrt[m]{\sqrt[n]{a}}=\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}}=a^{\frac{1}{n\cdot m}}=\sqrt[m\cdot n]{a}$ as $\sqrt[m]{\sqrt[n]{a}}=\sqrt[m\cdot n]{a}$

6. Two different surds can not be added unless you calculate their decimal approximations. Only alike surds can be added.

Cases

- For example: $\sqrt{3}+\sqrt{2}$ or $\sqrt{7}+\sqrt[3]{7}$ can only be solved by approximation or they should remain unsolved.
- $7\sqrt{5}+11\sqrt{5}-\sqrt{5}$ certainly can be simplified to $17\sqrt{5}$
- Sometimes the possibility that a surds addition can be simplified is hidden. For example:
  $\sqrt{8}+\sqrt{18}+\sqrt[4]{2500}=\sqrt{2^3}+\sqrt{2\cdot 3^2}+\sqrt[4]{2^2\cdot 5^4}=2\sqrt{2}+3\sqrt{2}+5\sqrt{2}=10\sqrt{2}$

7. **Rationalising the denominator.** Sometimes it is useful to get rid of surds from the bottom of a fraction. To do so, you have to multiply both numerator and denominator by a suitable expresion.

Examples

- $\frac{1}{\sqrt[3]{25}}=\frac{1}{\sqrt[3]{5^2}}=\frac{\sqrt[3]{5}}{\sqrt[3]{5^2}\cdot\sqrt[3]{5}}=\frac{\sqrt[3]{5}}{5}$
- $\frac{1}{5-\sqrt{3}}=\frac{5+\sqrt{3}}{(5-\sqrt{3})(5+\sqrt{3})}=\frac{5+\sqrt{3}}{5^2-(\sqrt{3})^2}=\frac{5+\sqrt{3}}{22}$

Notes: $\boxed{\sqrt{a}\cdot\sqrt{a}=(\sqrt{a})^2=\sqrt{a^2}=a}$ $\boxed{(a+\sqrt{b})\cdot(a-\sqrt{b})=a^2-b}$ $\boxed{(\sqrt{a}+\sqrt{b})\cdot(\sqrt{a}-\sqrt{b})=a-b}$ $\boxed{\sqrt[3]{a}\cdot\sqrt[3]{a^2}=a}$

EXERCISES

27) Write $x\sqrt{x}$ as x^q where $q \in Q$.

28) Write each of these in the form a^q where $q \in Q$:

(i) $a^7 \times a^2$

(ii) $\frac{a^{14}}{a^2}$

(iii) $(a^7)^3$

(iv) $\sqrt{a}$

(v) $\sqrt[3]{a}$

(vi) $\frac{1}{a^3}$

(vii) $\frac{a^{11} \times a^7}{a^3}$

(viii) $\frac{a^2}{\sqrt{a}}$

(ix) $a\sqrt{a}$

(x) $\frac{a^2}{a^{-3}}$

29) Write each of these as x^p, where $p \in Q$.

(i) $\sqrt[4]{x}$

(ii) $\frac{x}{\sqrt{x}}$

(iii) $\frac{x^3}{\sqrt{x}}$

(iv) $\frac{(x^{-3})(x^4)}{(x^{-7})}$

(v) $(x\sqrt{x})^3$

(vi) $\sqrt{x}\sqrt[3]{x}$

(vii) $\sqrt[3]{x^2}$

(viii) $\sqrt{x^3}$

(ix) $\sqrt{x^6}$

(x) $\frac{(x^5)(x^{-7})}{x^4}$

30) Write each of these as x^p, where $p \in Q$:

a) $\left(\sqrt[3]{x^2}\right)^5$ b) $\sqrt[15]{x^6} =$ c) $\sqrt{\frac{a^{13}}{a^6}} =$ d) $\sqrt[3]{\sqrt{x}} =$ e) $\sqrt[n]{\sqrt[m]{a^k}} =$

31) a) Write $8\sqrt{2}$ in the form 2^n where $n \in Q$.

b) Write $\frac{27\sqrt{3}}{3}$ in the form 3^p where $p \in Q$.

c) Write $\frac{4\sqrt{2}}{32}$ in the form 2^q where $q \in Q$.

d) Write $\frac{49\sqrt{7}}{\sqrt[3]{7}}$ in the form 7^p where $p \in Q$.

e) Write $\sqrt{49x^2y^6}$ in the form kx^py^q where $k, p, q \in Q$.

f) Write $\frac{125^{2/3} \times 5^2}{25 \times \sqrt{5}}$ in the form 5^p, where $p \in Q$.

h) Write $\frac{49\sqrt{7}}{7(\sqrt[3]{7})}$ in the form 7^n where $n \in Q$.

g) Evaluate the following and write it in the form 2^n: $\frac{64^{5/6} \times 8^{2/3}}{(\sqrt{2})^4 \times 4^{3/2}}$

32) Evaluate the following: a) $625^{\frac{}{4}} =$ b) $64^{\frac{5}{6}} =$

33) Evaluate the following:

1. $9^{1/2}$
2. $25^{1/2}$
3. 5^{-2}
4. $8^{1/3}$
5. 6^{-1}
6. 2^{-3}
7. $16^{1/2}$
8. $16^{-1/2}$
9. 3^{-1}
10. 3^{-3}
11. $\left(\frac{1}{9}\right)^{1/2}$
12. $100^{-1/2}$
13. $\left(\frac{4}{9}\right)^{1/2}$
14. $8^{2/3}$
15. $25^{3/2}$
16. $32^{4/5}$
17. $1000^{2/3}$
18. $100^{3/2}$
19. $9^{3/2}$
20. $8^{-2/3}$
21. $27^{-2/3}$
22. $(3\sqrt{2})^2$
23. 7^0
24. $81^{3/4}$
25. $\left(\frac{25}{49}\right)^{-1/2}$
26. $\left(\frac{1}{16}\right)^{-1/2}$
27. $\left(\frac{1}{27}\right)^{-1/3}$
28. $\left(\frac{81}{100}\right)^{-1/2}$
29. $\left(\frac{8}{27}\right)^{-1/3}$
30. $\left(\frac{27}{64}\right)^{-2/3}$
31. $\left(\frac{49}{100}\right)^{3/2}$
32. $\left(\frac{49}{100}\right)^{-3/2}$
33. $(2\tfrac{1}{4})^{-1/2}$
34. $\left(\frac{4}{7}\right)^{-1}$
35. $(1\tfrac{1}{2})^{-1}$
36. $\left(\frac{121}{100}\right)^{-1/2}$
37. $\left(\frac{4}{25}\right)^{-3/2}$
38. $\left(\frac{1}{2}\right)^{0}$
39. $\left(\frac{5}{12}\right)^{-1}\left(\frac{3}{2}\right)^{2}$
40. $(6\tfrac{1}{4})^{1/2}$
41. $2^2 + 16^{1/2}$
42. $2^5 - 5^2$
43. $4^{1/2} + 4^0$
44. $9^2 + 9^{1/2} + 9^0$
45. $100^{1/2} + 121^{1/2}$

34) Put <, = or > between each pair.

(i) $3^4 \ldots 4^3$

(ii) $2^4 \ldots 4^2$

(iii) $81^{1/2} \ldots (25^{1/2} + 25^{1/2})$

(iv) $(3^2 + 4^2) \ldots 5^2$

(v) $5^1 \ldots 1^5$

(vi) $0^2 \ldots 2^0$

(vii) $(2^7)^3 \ldots 2^7 \times 2^3$

(viii) $\frac{2^{12}}{2^2} \ldots 2^6$

(ix) $10^0 \ldots 0$

(x) $7 \times 7^3 \ldots 7^4$

(xi) $4^{1/2}$ 4^{-2}

(xii) $\sqrt{(4)^3}$ $(\sqrt{4})^3$

(xiii) $\left(\frac{1}{4}\right)^{-1}$ $\left(\frac{1}{4}\right)^{-2}$

35) Are these true or false?

(i) $(x^3)^2 = x^5$

(ii) $x^3 = \frac{x^6}{x^2}$

(iii) $x\sqrt{x} = (x^{1/2})^3$

(iv) $(2^2)(3^3) = 6^5$

36) If $x = 16$ and $y = 9$ investigate

$x^{1/2} + y^{1/2} = (x+y)^{1/2}$

37) Write these as surds:

a) $x^{\frac{7}{9}} =$ b) $(m^5 \cdot n^5)^{\frac{1}{3}} =$ c) $a^{\frac{1}{2}} \cdot b^{\frac{1}{3}} =$ d) $\left[(x^2)^{\frac{1}{3}}\right]^{\frac{1}{5}} =$

38) Solve for x:

1. $2^x = 4^2$
2. $3^x = 9\sqrt{3}$
3. $3^x(3^7) = 3^{21}$
4. $2^{x+1} = 16\sqrt{2}$
5. $5^{x-1} = \frac{25}{\sqrt{5}}$
6. $(2^x)^3 = (2^7)(2^5)$
7. $4^x = 8$
8. $25^x = 5$
9. $27^x = 9$
10. $1000^x = 10$
11. $4^x = ½$
12. $3^{x+2} = 81$
13. $8^x = ¼$
14. $32^x = 8$
15. $9^x = \frac{1}{27}$
16. $4(2^x) = \frac{16}{\sqrt{2}}$
17. $11^x = (121)^3$
18. $\frac{5^x}{\sqrt{5}} = 125$
19. $4^{x+1} = 2^{x-2}$
20. $9^{2x+1} = 3^{x-2}$

39) a) Find x and y if $3^{x+y} = 9$ and $2^{x-y} = 16$.

b) Find x and y if $9^x = 27$ and $y = (100)^x$.

40) a) Solve the simultaneous equations:

$3^{x+2y} = 1$ and $\frac{2^x}{2^y} = 8$

b) Solve the simultaneous equations:

$7^{x+2y} = \frac{1}{7}$ and $4^x = \frac{1}{8^y}$

41) Simplify $(x^{1/2} - y^{1/2})(x^{1/2} + y^{1/2})$ and hence evaluate $(x^{1/2} - y^{1/2})(x^{1/2} + y^{1/2})$ when $x = 7$ and $y = 3$.

42) Evaluate:

(i) $\sqrt{144}$

(ii) $2\sqrt{49}$

(iii) $\sqrt{900}$

(iv) $5\sqrt{400}$

(v) $\sqrt{9/16}$

43) Reduce these surds:

a) $\sqrt[12]{x^9} =$ b) $\sqrt[12]{x^8} =$ c) $\sqrt[5]{y^{10}} =$ d) $\sqrt[6]{8} =$

e) $\sqrt[9]{64} =$ f) $\sqrt[8]{81} =$

44) Take as many factors as possible out of the following:

a) $\sqrt[3]{32x^4} =$ b) $\sqrt[3]{81a^3b^5c} =$ c) $\sqrt[5]{64} =$

45) Reduce these surds:

(i) $\sqrt{20}$

(ii) $\sqrt{200}$

(iii) $\sqrt{63}$

(iv) $\sqrt{75}$

(v) $\sqrt{8}$

46) Simplify each of these as much as possible: a) $\sqrt{18} + \sqrt{50} - \sqrt{2} - \sqrt{8} =$ b) $\sqrt{50a} - \sqrt{18a} =$

47) Simplify each of these as much as possible:

(i) $\sqrt{27} - \sqrt{3}$

(ii) $\sqrt{24} + \sqrt{54}$

(iii) $\sqrt{98} - \sqrt{72}$

(iv) $\sqrt{28} + 2\sqrt{63} - \sqrt{7}$

(v) $3\sqrt{44} - \sqrt{99}$

(vi) $\sqrt{8} + \sqrt{18}$

(vii) $\sqrt{32} + \sqrt{200}$

(viii) $\sqrt{300} - \sqrt{75}$

(ix) $\sqrt{2} + \sqrt{50} - \sqrt{72}$

(x) $\sqrt{40} + \sqrt{90} - \sqrt{160}$

48) Simplify: a) $\sqrt[3]{2} \cdot \sqrt[5]{2} =$ b) $\sqrt[3]{9} \cdot \sqrt[6]{3} =$ c) $\sqrt[10]{a^4 \cdot b^6} =$

49) Simplify: a) $\dfrac{\sqrt{9}}{\sqrt[3]{3}} =$ b) $\dfrac{\sqrt[5]{16}}{\sqrt{2}} =$ c) $\dfrac{\sqrt[4]{a^3b^5c}}{\sqrt{ab^3c^3}} =$ d) $\left(\sqrt[3]{a^2}\right)^6 =$

e) $\left(\sqrt{x}\right)^3 \cdot \sqrt[3]{x} =$ f) $\left(\sqrt{\sqrt{\sqrt{2}}}\right)^8 =$

50) Simplify:

(i) $(2\sqrt{3})(5\sqrt{3})$

(ii) $(\sqrt{2})(\sqrt{8})$

(iii) $(\sqrt{2})(\sqrt{10})$

(iv) $(\sqrt{7})(\sqrt{14})$

(v) $(2\sqrt{6})(2\sqrt{15})$

(vi) $2\sqrt{3}(3\sqrt{3} - 1)$

(vii) $\sqrt{7}(3\sqrt{2} - 5\sqrt{7})$

(viii) $2\sqrt{2}(4\sqrt{2} - \sqrt{5})$

(ix) $\sqrt{2}(2\sqrt{32} - \sqrt{50})$

(x) $(\sqrt{5} - 1)(\sqrt{5} + 1)$

51) a) Simplify $(\sqrt{8}-1)(\sqrt{2}+3)$

b) Simplify $\dfrac{(3+\sqrt{5})(3-\sqrt{5})}{2}$

c) Evaluate $\dfrac{(\sqrt{11}+\sqrt{3})(\sqrt{11}-\sqrt{3})}{4}$

52) State if these are true or false:

(i) $\sqrt{3}+\sqrt{7} = \sqrt{10}$

(ii) $(\sqrt{11})(\sqrt{3}) = \sqrt{33}$

(iii) $\sqrt{14}-\sqrt{6} = \sqrt{8}$

(iv) $\dfrac{\sqrt{14}}{\sqrt{2}} = \sqrt{7}$

53) Put <, > or = in each of the gaps:

(i) $\sqrt{2}\times\sqrt{3}$ .... $\sqrt{6}$

(ii) $\frac{1}{2}\sqrt{20}$ .... $\sqrt{10}$

(iii) $\sqrt{16}+\sqrt{9}$ .... $\sqrt{16+9}$

(iv) $\sqrt{3}+\sqrt{12}$ .... $\sqrt{27}$

(v) $\frac{1}{4}$ .... $\sqrt{\frac{1}{4}}$

(vi) $\sqrt[3]{51}$ .... $\sqrt[9]{132650}$

(vii) $\sqrt[4]{31}$ .... $\sqrt[3]{13}$

54) Rationalise: a) $\dfrac{5}{\sqrt[3]{2}} =$ b) $\dfrac{\sqrt{5}}{\sqrt{7}} =$ c) $\dfrac{3}{2-\sqrt{3}} =$ d) $\dfrac{4}{\sqrt{3}+\sqrt{2}} =$ e) $\dfrac{1}{\sqrt[5]{3^2}} =$

55) Write $(\sqrt{3}+2)^2$ in the form $a+b\sqrt{3}$, where $a, b \in N$.

56) a) Use the formula $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to find the roots of $x^2-4x+1 = 0$ in the form $a\pm\sqrt{b}$, where $a, b \in N$.

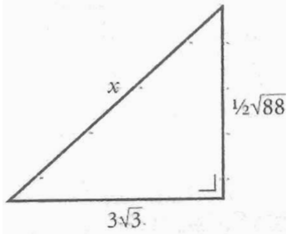
b) Find the roots of $x^2-2x-6 = 0$ in the form $a\pm\sqrt{b}$, where $a, b \in Z$.

c) (i) Find the roots of $x^2-6x+7 = 0$ in the form $a\pm\sqrt{b}$, where $a, b \in Z$.

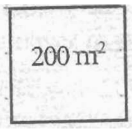
(ii) Find the sum of the two roots.

(iii) Find the product of the two roots.

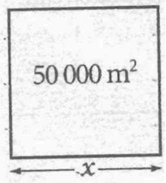
57) Use Pythagoras' Theorem to find the value of x in the diagram.



58) a) The area of a square is 200 m^2. Find the length of each side, correct to the nearest centimetre.



b) The area of a square field is 50 000 m^2. Find the length of each side, correct to the nearest metre.



59) a) If $a = 79.5$ and $b = 33.4$ investigate if $\sqrt{a-b} = \sqrt{a} - \sqrt{b}$.
(Take square roots correct to 3 decimal places.)

b) If $K = 75.35$, investigate if $\sqrt{2K} = 2\sqrt{K}$.
(Take square roots correct to two decimal places.)

c) If $a = 1369$ and $b = 1296$,
(i) find the values of $\sqrt{a}$, $\sqrt{b}$ and $\sqrt{a-b}$.
(ii) show that $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$

d) Given that $x = 3969$ and $y = 256$,
(i) find the value of $\sqrt{x}$, $\sqrt{y}$, $\sqrt{x+y}$
(ii) investigate if $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

e) Given that $x = 21.46$, $y = 33.12$,
(i) find (correct to 3 decimal places) the values of $\sqrt{x}$, $\sqrt{y}$, $\sqrt{x+y}$
(ii) investigate if $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

60) Factorise the rooted and evaluate:

a) $\sqrt{324}$ b) $\sqrt{441}$ c) $\sqrt{1024}$ d) $\sqrt[3]{64}$ e) $\sqrt[4]{625}$ f) $\sqrt[3]{-1}$ g) $\sqrt[3]{-1000}$ h) $\sqrt[4]{10000}$
i) $\sqrt[5]{-32}$ k) $\sqrt[5]{243}$ l) $\sqrt[5]{100000}$ m) $\sqrt[10]{1024}$ n) $\sqrt{0'49}$ o) $\sqrt{0'04}$ p) $\sqrt{1'44}$ q) $\sqrt{0'0009}$

61) Evaluate:

a) $5\sqrt{4}-6\sqrt{9}-7\sqrt{36}$ b) $7+3\sqrt[3]{-8}$ c) $\sqrt{10+2\sqrt{7+\sqrt[3]{8}}}$ d) $\sqrt{2\cdot\sqrt[5]{4\cdot\sqrt{64}}}$ e) $\sqrt{4'56-5\sqrt{0'16}}$

62) Evaluate:

a) $\sqrt{225\cdot 196\cdot 16}$ b) $\sqrt{14400}$ c) $\sqrt{225{:}25}$ d) $\sqrt{49{:}0'0001}$ e) $\sqrt{6'25{:}0'01}$ f) $\sqrt{\frac{1}{4}}-\sqrt{\frac{9}{25}}$ g) $2\sqrt{\frac{9}{4}}+\sqrt{\frac{121}{100}}$

63) Simplify the following:

a) $6\sqrt{3}-4\sqrt{3}+5\sqrt{3}$ b) $3\sqrt{2}-3\sqrt{8}+3\sqrt{18}$ c) $7\sqrt{18}-3\sqrt{32}-\frac{3}{5}\sqrt{50}$

64) Simplify the following:

a) $\sqrt{8}\cdot\sqrt{5}\cdot\sqrt{10}$ b) $\sqrt{3}\cdot\sqrt{8}\cdot\sqrt{30}\cdot\sqrt{20}$ c) $\sqrt{12}\cdot\sqrt{\frac{3}{4}}\cdot\sqrt{\frac{12}{5}}\cdot\sqrt{\frac{15}{4}}$ d) $\sqrt[3]{\frac{5}{9}}\cdot\sqrt[3]{\frac{9}{2}}\cdot\sqrt[3]{\frac{25}{4}}$
e) $\sqrt{50}{:}\sqrt{18}$ f) $(\sqrt{2}+\sqrt{5})^2$ g) $(\sqrt{2}+\sqrt{5})(\sqrt{2}-\sqrt{5})$ h) $(\sqrt{2}-\sqrt{5})^2$

65) Reduce the following:

a) $\sqrt[3]{b^6}$ b) $\sqrt[4]{b^6}$ c) $\sqrt{a^9}$ d) $\sqrt{b^{15}}$ e) $\sqrt{27a^6}$
f) $\sqrt{216}$ g) $\sqrt[20]{1024}$ h) $\sqrt[3]{162000}$ i) $\sqrt{81\cdot a^6\cdot b^5\cdot c^8}$ k) $\sqrt[12]{729}$
l) $\sqrt[3]{\sqrt{7}}$ m) $\sqrt{2\sqrt{2\sqrt{2}}}$ n) $\sqrt{\sqrt[4]{512\cdot a^8 b^{10}}}$ ñ) $\left(\sqrt{\sqrt{\sqrt{\sqrt{16x^2}}}}\right)^2$

66) Rationalise and simplify:

a) $\frac{3\sqrt{2}}{\sqrt{3}}$ b) $\frac{4+3\sqrt{6}}{2\sqrt{6}}$ c) $\frac{5\sqrt{7}}{7\sqrt{5}}$ d) $\frac{2-\sqrt{5}}{3-\sqrt{7}}$ e) $\frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}+\sqrt{6}}$ f) $\frac{2\sqrt{5}}{\sqrt{5}+\sqrt{6}}$ g) $\frac{3+\sqrt{3}}{2\sqrt{3}+\sqrt{6}}$