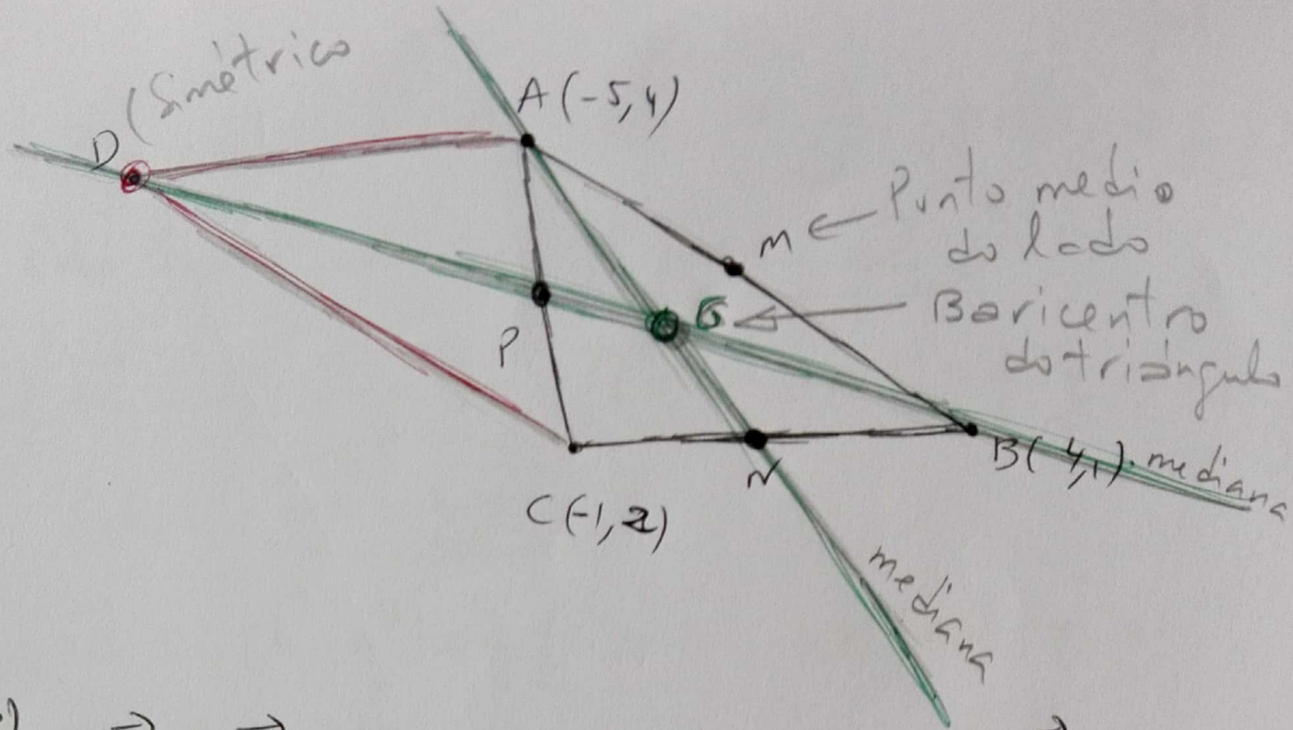


a) $\vec{AM} = \vec{MB}$; $\vec{m} - \vec{a} = \vec{b} - \vec{m}$, $2\vec{m} = \vec{a} + \vec{b}$

$$\vec{m} = \frac{\vec{a}+\vec{b}}{2} = \frac{(-5,4)+(4,1)}{2} = \frac{(-1,5)}{2} = \boxed{\left(-\frac{1}{2}, \frac{5}{2}\right)}$$

Da mesma maneira

$$\vec{n} = \frac{\vec{b}+\vec{c}}{2} = \frac{(4,1)+(-1,2)}{2} = \boxed{\left(\frac{3}{2}, \frac{3}{2}\right)}$$

$$\vec{p} = \frac{\vec{a}+\vec{c}}{2} = \frac{(-5,4)+(-1,2)}{2} = \boxed{(-3,3)}$$

b) $\vec{MN} = \vec{n} - \vec{m} = \left(\frac{3}{2}, \frac{3}{2}\right) - \left(-\frac{1}{2}, \frac{5}{2}\right) = \left(\frac{4}{2}, -\frac{2}{2}\right) = (2,-1)$

$$\boxed{\vec{MN} = (2,-1)}$$

$\vec{MP} = \vec{p} - \vec{m} = (-3,3) - \left(-\frac{1}{2}, \frac{5}{2}\right) = \left(-\frac{5}{2}, \frac{1}{2}\right)$

$$\boxed{\vec{MP} = \left(-\frac{5}{2}, \frac{1}{2}\right)}$$

$\vec{PN} = \vec{n} - \vec{p} = \left(\frac{3}{2}, \frac{3}{2}\right) - (-3,3) = \left(\frac{9}{2}, -\frac{3}{2}\right)$

$$\boxed{\vec{PN} = \left(\frac{9}{2}, -\frac{3}{2}\right)}$$

$\frac{1}{2}\vec{BC} = \frac{1}{2}(\vec{c}-\vec{b}) = \frac{1}{2}[(-1,2)-(4,1)] = \frac{1}{2}(-5,1) = (-\frac{5}{2},\frac{1}{2})$

Polo tanto coincide $\vec{MP} = \frac{1}{2}\vec{BC}$

$\frac{1}{2}\vec{AC} = \frac{1}{2}(\vec{c}-\vec{a}) = \frac{1}{2}[(-1,2)-(-5,4)] = \frac{1}{2}(4,-2) = (2,-1)$

Enton $\vec{MN} = \frac{1}{2}\vec{AC}$

$\frac{1}{2}\vec{AB} = \frac{1}{2}(\vec{b}-\vec{a}) = \frac{1}{2}[(4,1)-(-5,4)] = \frac{1}{2}(9,-3)$

$= (\frac{9}{2},-\frac{3}{2})$

Entón $\vec{PN} = \frac{1}{2}\vec{AB}$ Quedo comprobado.

c) recta que pasa por BP.

Vector director $\vec{d} = \vec{BP} = (-3,3)-(4,1) = (-7,2)$ (d_1, d_2)

Punto $B = (4,1)$ $(x_0\ y_0)$

Ec. paramétricas $\boxed{\begin{cases} x = 4-7t \\ y = 1+2t \end{cases}}$ $\begin{cases} x = x_0 + d_1 t \\ y = y_0 + d_2 t \end{cases}$

d) Recta que pasa por AN : $y - y_0 = m(x-x_0)$

x	y
-5	4
$\frac{3}{2}$	$\frac{3}{2}$

$m = \dfrac{\frac{3}{2}-4}{\frac{3}{2}+5} = -\dfrac{5}{13}$

$A = (-5,4)$ $x_0\ y_0$

$\boxed{y-4 = -\frac{5}{13}(x+5)}$

e) Punto intersección das rectas anteriores.
Resolvemos o sistema das súas ecuacións

$$\left.\begin{array}{l} x = 4-7t \\ y = 1+2t \\ y-4 = -\frac{5}{13}(x+5) \end{array}\right\}$$ Substituímos as dúas primeiras na terceira

$1+2t-4 = -\frac{5}{13}(4-7t+5)$; $-3+2t = -\frac{5}{13}(9-7t)$;

$(-3+2t).13 = -5(9-7t)$; $-39+26t = -45+35t$

$6 = 9t \rightarrow t = \frac{2}{3}$

$$\left.\begin{array}{l} x = 4-7.\frac{2}{3} \Rightarrow x = -\frac{2}{3} \\ y = 1+\frac{2}{3}.2 \Rightarrow y = \frac{7}{3} \end{array}\right\} \boxed{G = \left(-\frac{2}{3}, \frac{7}{3}\right)}$$

Punto intersección.

f) $\vec{AG} = \frac{2}{3}\vec{AN}$; $\vec{g}-\vec{a} = \frac{2}{3}(\vec{n}-\vec{a})$;

$3\vec{g}-3\vec{a} = 2\vec{n}-2\vec{a}$; $3\vec{g} = 3\vec{a}-2\vec{a}+2\vec{n}$

$3\vec{g} = \vec{a}+2\vec{n}$; $3\vec{g} = \vec{a}+\cancel{2}\frac{\vec{b}+\vec{c}}{\cancel{2}}$

$$\boxed{\vec{g} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}}$$

xa que $\vec{n} = \frac{\vec{b}+\vec{c}}{2}$

g) $\vec{g} = \frac{(-5,4)+(4,1)+(-1,2)}{3} = \frac{(-2,7)}{3} \Rightarrow G = \left(-\frac{2}{3}, \frac{7}{3}\right)$

Vemos que coincide co punto intersección das rectas medianas. A este punto chámaselle Baricentro ou Centro de Gravidade do triángulo.

h) Punto D simétrico

$\vec{PD} = \vec{BP}$; $\vec{d} - \vec{p} = \vec{p} - \vec{b}$; $\vec{d} = 2\vec{p} - \vec{b}$

$\vec{d} = 2(-3,3) - (4,1) = (-6,6) - (4,1) = (-10,5) \Rightarrow$

$\Rightarrow \boxed{D = (-10,5)}$

i) Perímetro cuadrilátero ABCD

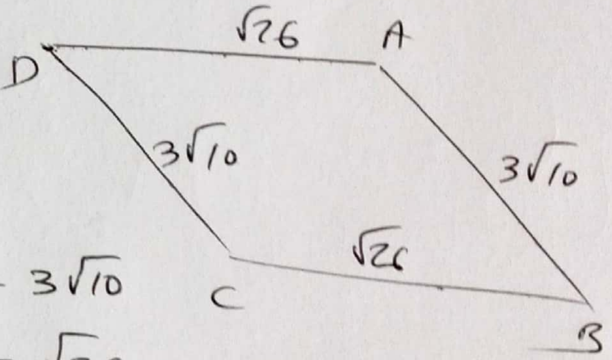
$\vec{AB} = (9,-3)$

$\vec{BC} = (-5,1)$

$\vec{DC} = (9,-3)$

$\vec{AD} = (-5,1)$

$\left.\begin{array}{l} \vec{AB} = \vec{DC} \\ \vec{BC} = \vec{AD} \end{array}\right\} \Rightarrow$ É un paralelogramo.



$|\vec{AB}| = |\vec{DC}| = \sqrt{(9)^2 + (-3)^2} = 3\sqrt{10}$

$|\vec{BC}| = |\vec{AD}| = \sqrt{(-5)^2 + (1)^2} = \sqrt{26}$

$\boxed{\text{Perímetro} = 2(3\sqrt{10} + \sqrt{26})}$