SISTEMAS NON LINEARES

a) $\left.\begin{array}{r} x^2+\cancel{y^2}+\cancel{xy}=\frac{3}{4} \\ + \quad x^2-\cancel{y^2}-\cancel{xy}=-\frac{1}{4} \end{array}\right\}$ Por reducción

$2x^2 = \frac{1}{2} \Rightarrow x^2=\frac{1}{4} \Rightarrow x=\pm\frac{1}{2}$

Para $x=\frac{1}{2} \to \left(\frac{1}{2}\right)^2+y^2+\frac{1}{2}y=\frac{3}{4}$; $\frac{1}{4}+y^2+\frac{1}{2}y=\frac{3}{4}$

$1+4y^2+2y=3$; $4y^2+2y-2=0$; $2y^2+y-1=0$

$y=\frac{-1\pm\sqrt{1^2-4\cdot2(-1)}}{2\cdot2}=\frac{-1\pm\sqrt{9}}{4}=\frac{-1\pm3}{4}= \begin{cases} y_1=\frac{1}{2} \\ y_2=-1 \end{cases}$

$$\boxed{\text{Solución } \begin{cases} \left(\frac{1}{2},\frac{1}{2}\right) \\ \left(\frac{1}{2},-1\right) \end{cases}}$$

Para $x=-\frac{1}{2} \to \left(-\frac{1}{2}\right)^2+y^2-\frac{1}{2}y=\frac{3}{4}$; $\frac{1}{4}+y^2-\frac{1}{2}y=\frac{3}{4}$;

$1+4y^2-2y=3$; $4y^2-2y-2=0$; $2y^2-y-1=0$

$y=\frac{1\pm\sqrt{1-4\cdot2(-1)}}{2\cdot2}=\frac{1\pm3}{4}= \begin{cases} y_3=1 \\ y_4=-\frac{1}{2} \end{cases}$

$$\boxed{\text{Solución } \begin{cases} \left(-\frac{1}{2},1\right) \\ \left(-\frac{1}{2},-\frac{1}{2}\right) \end{cases}}$$

Por tanto 4 solución

b) $x + y - \frac{y}{x} = 1 \rightarrow x^2 + xy - y = x$ } Por substitución
$x + y = 5 \rightarrow y = 5 - x$

$x^2 + x(5-x) - (5-x) = x$
$\cancel{x^2} + 5x - \cancel{x^2} - 5 + \cancel{x} - \cancel{x} = 0$; $5x = 5$; $x = 1$

$y = 5 - 1 \rightarrow y = 4$ $\boxed{\text{Solución } (1, 4)}$

c) $3x^2 - 5y^2 = 30$ } Por reducción
$x^2 - 2y^2 = 7$

$3x^2 - 5y^2 = 30$
$-3x^2 + 6y^2 = -21$
$y^2 = 9 \Rightarrow y = \pm 3$

* Para $y = 3$
$x^2 - 2(3)^2 = 7$; $x^2 = 25$; $x = \pm 5$

$\boxed{\text{Solución} \begin{cases} (-5, 3) \\ (5, 3) \end{cases}}$

Para $y = -3$
$x^2 - 2(-3)^2 = 7$; $x^2 = 25$; $x = \pm 5$

$\boxed{\text{Solución} \begin{cases} (-5, -3) \\ (5, -3) \end{cases}}$

d) $E_1: \frac{2x-1}{x+1} + \frac{y+3}{y+1} = 3$

$E_2: x(x-2) = y(1-y)$

Transformamos por álxebra o sistema noutro máis sinxelo

$E_1: (2x-1)(y+1) + (x+1)(y+3) = 3(x+1)(y+1)$

$2xy + 2x - \cancel{y} - 1 + xy + \cancel{y} + 3x + 3 = 3(xy + x + y + 1)$

$3xy + 5x + 2 = 3xy + 3x + 3y + 3$

$\cancel{3xy} + 5x + 2 - \cancel{3xy} - 3x - 3y - 3 = 0 \rightarrow E_1: 2x - 3y = 1$

$E_2: x^2 - 2x = y - y^2 \rightarrow x^2 + y^2 - 2x - y = 0 : E_2$

$\left.\begin{array}{l} 2x - 3y = 1 \\ x^2 + y^2 - 2x - y = 0 \end{array}\right\} \rightarrow x = \frac{1+3y}{2}$ Por substitución

$\left(\frac{1+3y}{2}\right)^2 + y^2 - \cancel{2}\frac{(1+3y)}{\cancel{2}} - y = 0$

$\frac{1+6y+9y^2}{4} + y^2 - 1 - 3y - y = 0$; $1 + 6y + 9y^2 + 4y^2 - 4 - 12y - 4y = 0$

$13y^2 - 10y - 3 = 0$; $y = \frac{10 \pm \sqrt{100 + 4.13.3}}{2.13} = \frac{10 \pm 16}{26} =$

$= \begin{cases} y_1 = 1 \rightarrow x_1 = \frac{1+3.1}{2} \Rightarrow x_1 = 2 \\ y_2 = -\frac{3}{13} \rightarrow x_2 = \frac{1 - 3 \cdot \frac{3}{13}}{2} = \frac{2}{13} = x_2 \end{cases}$

$\boxed{\text{Solución: } \begin{cases} (2,1) \\ \left(\frac{2}{13}, -\frac{3}{13}\right) \end{cases}}$

e) $E_1: \; x y = 12$

$E_2: \; x + y + \sqrt{x^2+y^2} = 12 \rightarrow \sqrt{x^2+y^2} = 12 - x - y$

$E_2: \left(\sqrt{x^2+y^2}\right)^2 = (12 - x - y)^2$

$x^2 + y^2 = 144 + x^2 + y^2 - 24x - 24y + 2xy$

$\cancel{x^2} + \cancel{y^2} - \cancel{x^2} - \cancel{y^2} + 24x + 24y - 2xy - 144 = 0$

$E_2: \; 12x + 12y - xy - 72 = 0$

$E_1: \; xy = 12$

Por substitución

$\rightarrow x = \frac{12}{x}$

$12 \cdot \frac{12}{y} + 12y - \frac{12}{\cancel{y}} \cdot \cancel{y} - 72 = 0$ Dividimos por 12

$\frac{12}{y} + y - 7 = 0 \rightarrow 12 + y^2 - 7y = 0; \; y^2 - 7y + 12 = 0$

$y = \frac{7 \pm \sqrt{49-48}}{2} = \frac{7 \pm 1}{2} = \begin{cases} y_1 = 4 \rightarrow x_1 = \frac{12}{4} = 3 \\ y_2 = 3 \rightarrow x_2 = \frac{12}{3} = 4 \end{cases}$

Solución $\begin{cases} (3,4) \\ (4,3) \end{cases}$

8) $E_1: \dfrac{3}{x} + \dfrac{y+2}{y} = 1$
$E_2: xy + 1 = -5$

$\rightarrow 3y + x(y+2) = xy$
$3y + xy + 2x = xy$
$2x + 3y + \cancel{xy} - \cancel{xy} = 0$

$E_1: 2x+3y = 0$
$E_2: xy+1 = -5$

$\rightarrow x = -\dfrac{3y}{2}$ Por substitución

$-\dfrac{3y}{2} \cdot y + 1 = -5; -\dfrac{3y^2}{2} = -6$

$3y^2 = 12; \; y^2 = 4; \; y = \pm 2$

$y_1 = -2 \rightarrow x_1 = -\dfrac{3(-2)}{2} \Rightarrow x_1 = 3$
$y_2 = 2 \rightarrow x_2 = -\dfrac{3(2)}{2} \Rightarrow x_1 = -3$

$\boxed{\text{Solución } \begin{cases}(3,-2)\\(-3,2)\end{cases}}$

9) $E_1: \sqrt{x+6} = y+1$
$E_2: 2x - y = -5$

$\rightarrow (\sqrt{x+6})^2 = (y+1)^2$
$x+6 = y^2+2y+1; \; y^2+2y-x-5=0$

$E_1: y^2+2y-x-5=0$
$E_2: -y+2x+5=0$

Reducimos a "x"

$$\begin{array}{r} 2y^2+4y-\cancel{2x}-10=0 \\ -y+\cancel{2x}+5=0 \\ \hline 2y^2+3y \quad -5=0 \end{array}$$

$y = \dfrac{-3 \pm \sqrt{9+40}}{4} = \dfrac{-3\pm 7}{2} =$

$= \begin{cases} y_1 = 1 \rightarrow 2x-1=-5, \; x_1 = -2 \\ \cancel{y_2 = -5/2 \rightarrow 2x + \frac{5}{2} = -5, \; x_2 = -\frac{15}{4}} \end{cases}$

$\boxed{\text{Solución } (-2,1)}$

$\cancel{\left(-\dfrac{15}{4}, -\dfrac{5}{2}\right)}$ Non satisfai a E_1

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h) $E_1: \sqrt{3-x} = y+2$
$E_2: x+1 = \sqrt{2y}$

Por igualación

Despexamos "x" nas dúas ecuacións

$E_1: (\sqrt{3-x})^2 = (y+2)^2 \rightarrow 3-x = y^2+4y+4$

$-y^2-4y-1 = x$

$E_2: x = \sqrt{2y}-1$

$\sqrt{2y}\cancel{-1} = -y^2-4y\cancel{-1}$

$(\sqrt{2y})^2 = (-y^2-4y)^2$

$2y = y^4+8y^3+16y^2$

$y^4+8y^3+16y^2-2y=0 \rightarrow y(y^3+8y^2+16y-2)=0$

$\rightarrow y_1 = 0$

$\rightarrow y^3+8y^2+16y-2=0 \rightarrow y_2 \simeq 0{,}118$

	1	8	16	-2
0,118		0,118	0,9579	2
	1	8,118	16,9579	0

$y^2+8{,}118y+16{,}9579=0$

$D = 8{,}118^2-4 \cdot 16{,}9579 < 0$

$\boxed{\text{Soluc. } (-1, 0)}$

$y_1 = 0 \rightarrow x_1 = \sqrt{2 \cdot 0}-1 \rightarrow x_1 = -1$

$y_2 \simeq 0{,}118 \rightarrow x_2 \simeq \cancel{\sqrt{2 \cdot 0{,}118}-1} \rightarrow x_2 \simeq -0{,}514$ Non cumple a E_1