SOLUCIÓNS

1) a) $\log_3 \frac{1}{9} - \log_3 \sqrt{3} + \log_3 81 = \log_3 3^{-2} - \log_3 3^{\frac{1}{2}} + \log_3 3^4 = -2 - \frac{1}{2} + 4 = \boxed{\frac{3}{2}}$

b)

a) $\frac{\sqrt{3}}{\sqrt{3}-1} = \frac{\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)\cdot(\sqrt{3}+1)} = \frac{3+\sqrt{3}}{(\sqrt{3})^2 - 1^2} = \frac{3+\sqrt{3}}{3-1} = \boxed{\frac{3+\sqrt{3}}{2}}$

b) $\sqrt{45} - \sqrt{20} + \sqrt{90} = \sqrt{3^2 \cdot 5} - \sqrt{2^2 \cdot 5} + \sqrt{3^2 \cdot 2 \cdot 5} = 3\sqrt{5} - 2\sqrt{5} + 3\sqrt{10} =$
$= \boxed{\sqrt{5} + 3\sqrt{10}}$

2)

a) $\frac{1}{x+1} : \frac{1}{2x-1} - \frac{1}{x^2-1} = \frac{2x-1}{x+1} - \frac{1}{x^2-1} = \frac{2x-1}{x+1} - \frac{1}{(x+1)(x-1)} =$

$= \frac{(2x-1)\cdot(x-1) - 1}{(x+1)\cdot(x-1)} = \frac{2x^2-2x-x+1-1}{(x+1)\cdot(x-1)} = \boxed{\frac{2x^2-3x}{x^2-1}}$

b) $1 - x = \sqrt{7-3x}$

$(1-x)^2 = (\cancel{\sqrt{}}7-3x)^{\cancel{2}}$

$1 - 2x + x^2 = 7 - 3x$

$x^2 + x - 6 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1}$

$x = \frac{-1 \pm 5}{2} \begin{cases} x_1 = 2 \\ x_2 = -3 \end{cases}$

PROBA (hai que facela, pois aparece unha $\sqrt{\ }$)

$x = 2$

$1 - 2 = \sqrt{7-3\cdot 2}$

$-1 \neq 1$ non vale

$\boxed{x=-3}$

$1-(-3) = \sqrt{7-3(-3)}$ si vale

$4 = 4$

Solución: $x = -3$

c) $\left.\begin{aligned} x - y + 2z &= -1 \\ 2x + y + z &= 1 \\ x + y + z &= 2 \end{aligned}\right\}$ $\begin{matrix} \\ E_2 - 2E_1 \\ E_3 - E_1 \end{matrix}$ $\left.\begin{aligned} x - y + 2z &= -1 \\ 3y - 3z &= 3 \\ 2y - z &= 3 \end{aligned}\right\}$ $E_2 - 3E_3$ $\left.\begin{aligned} x - y + 2z &= -1 \\ 3y - 3z &= 3 \\ -3y \quad &= -6 \end{aligned}\right]$ $\to \boxed{y=2}$

$3\cdot 2 - 3z = 3$

$-3z = -3$

$\boxed{z=1}$

$x - 2 + 2 = -1$

$\boxed{x=-1}$

Solución: $\boxed{x=-1,\ y=2,\ z=1}$

3) a) $\left.\begin{aligned} 2x - 6 &< 0 \\ x - 5 &\leq 2x - 1 \end{aligned}\right\}$

$2x - 6 < 0$

$2x < 6$

$x < 3$

$S_1 = (-\infty, 3)$

$x - 5 \leq 2x - 1$

$-x \leq 4$

$x \geq -4$

$S_2 = [-4, \infty)$

$-4 \qquad 3$

Solución: $[-4, 3)$

b) $\left.\begin{aligned} 3x - 1 &> y \\ x - 3 &< 2y \end{aligned}\right\}$

$3x - 1 = y$

x	y
0	-1
1	2
2	5

$P(0,0)$

$3\cdot 0 - 1 > 0$

$-1 > 0$ non

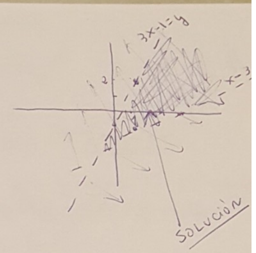
$x - 3 = 2y$

x	y
3	0
5	1
1	-1

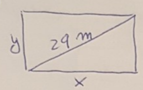
$P(0,0)$

$0 - 3 < 2\cdot 0$

$-3 < 0$ si



4)



$S = 420\ m^2$

$\left.\begin{aligned} xy &= 420 \\ x^2 + y^2 &= 29^2 \end{aligned}\right\}$ $\left.\begin{aligned} xy &= 420 \\ x^2 + y^2 &= 841 \end{aligned}\right\}$ $y = \frac{420}{x}$

$x^2 + \left(\frac{420}{x}\right)^2 = 841$

$x^2 + \frac{176400}{x^2} = 841$

$x^4 + 176400 = 841x^2$

$x^4 - 841x^2 + 176400 = 0$

$\boxed{z = x^2}$

$z^2 - 841z + 176400 = 0$

$z = \frac{841 \pm \sqrt{841^2 - 4\cdot 1\cdot 176400}}{2\cdot 1}$

$z = \frac{841 \pm 41}{2} \begin{cases} z_1 = 441 \\ z_2 = 400 \end{cases}$

Si $z = 441$

$x^2 = 441$

$x = \sqrt{441} = \pm 21$

Si $z = 400$

$x^2 = \sqrt{400} = \pm 20$

Si $x = 21 \Rightarrow y = \frac{420}{21} = 20$

Si $x = 20 \Rightarrow y = \frac{420}{20} = 21$

Aínda que o sistema ten varias solucións, no contexto do proble[ma] é a mesma, $x = 21,\ y = 20$

Solución: necesitaremos $2\cdot(21+20) = 2\cdot 41 = 82$ m de valla

5) $C = 19500$€

$n = 42$

$r = 7'5$

$m = C\,\frac{(1+i)^n \cdot i}{(1+i)^n - 1}$

$i = \frac{r}{1200} = \frac{7'5}{1200}$

$m = 19500 \cdot \frac{\left(1 + \frac{7'5}{1200}\right)^{42} \cdot \frac{7'5}{1200}}{\left(1 + \frac{7'5}{1200}\right)^{42} - 1} \simeq 529'33$€

6) $r = 3'6\%$

TAE $\to$ $\left(1 + \frac{3'6}{1200}\right)^{12} \simeq 1'0366 \to$ TAE $\simeq 3'66\%$ periodos mensu[ais]

$\left(1 + \frac{3'6}{400}\right)^{4} \simeq 1'0365 \to$ TAE $\simeq 3'65\%$ " trime[strais]

$\left(1 + \frac{3'6}{200}\right)^{2} \simeq 1'0363 \to$ TAE $\simeq 3'63\%$ " sem[estrais]

7) $f(x) = 300 - 6x$

$x =$ prezo do billete

$f(x) =$ número de viaxeiros

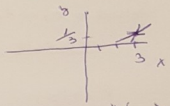
a) $I(x) =$ prezo · nº de viaxeiros $= x(300-6x) = \underline{\underline{300x - 6x^2}}$

b) é unha función cuadrática, graficamente é unha parábola cóncava ($a=-6<0$), polo tanto o punto máximo é o vèrtice da parábola.

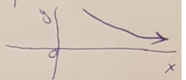
$$V_x = \frac{-b}{2a} = \frac{-300}{2\cdot(-6)} = 25$$

$$V_y = I(25) = 300\cdot 25 - 6\cdot 25^2 = 3750$$

$\boxed{\text{Solución}}$: o billete debe custar $\underline{\underline{25€}}$ para que os ingresos máximos sexan $\underline{\underline{3750€}}$



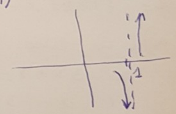
8) a) $\lim\limits_{x\to 3} \dfrac{2x-2}{x^3-2x^2+x} = \dfrac{2\cdot 3-2}{3^3-2\cdot 3^2+3} = \dfrac{4}{12} = \boxed{\dfrac{1}{3}}$



b) $\lim\limits_{x\to +\infty} \dfrac{2x-2}{x^3-2x^2+x} = \lim\limits_{x\to+\infty} \dfrac{2x}{x^3} = \lim\limits_{x\to+\infty} \dfrac{2}{x^2} = \dfrac{2}{\infty} = 0$

c) $\lim\limits_{x\to 1} \dfrac{2x-2}{x^3-2x^2+x} = \dfrac{2\cdot 1-2}{1^3-2\cdot 1^2+1} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 1} \dfrac{2\cancel{(x-1)}}{x\cdot(x-1)^{\cancel{2}}} =$

$= \lim\limits_{x\to 1} \dfrac{2}{x(x-1)} = \dfrac{2}{0} \Rightarrow \begin{cases} \lim\limits_{x\to 1^+} \dfrac{2}{x(x-1)} = +\infty \\ \lim\limits_{x\to 1^-} \dfrac{2}{x(x-1)} = -\infty \end{cases}$



9) • $f(x) = \dfrac{x^2-3}{2x^3+1}$ (regra do cociente)

$f'(x) = \dfrac{2x(2x^3+1) - 6x^2(x^2-3)}{(2x^3+1)^2} = \dfrac{4x^4+2x-6x^4+18x^2}{(2x^3+1)^2} = \dfrac{-2x^4+18x^2+2x}{(2x^3+1)^2}$

• $f(x) = \ln(x^4-2x)$ (derivada de $\ln f$)

$f'(x) = \dfrac{1}{x^4-2x}\cdot(4x^3-2) = \dfrac{4x^3-2}{x^4-2x}$

• $f(x) = \sqrt{2x^3-3} = (2x^3-3)^{\frac{1}{2}}$ (derivada de f^n)

$f'(x) = \frac{1}{2}(2x^3-3)^{\frac{1}{2}-1}\cdot 6x^2 = \frac{1}{2}(2x^3-3)^{-\frac{1}{2}}\cdot 6x^2 = \dfrac{3x^2}{\sqrt{2x^3-3}}$

• $f(x) = e^x\cdot \operatorname{sen} x$ (regra do producto)

$f'(x) = e^x\cdot\operatorname{sen}x + e^x\cdot\cos x$

10.- $f(x) = 2800 - 80x$ nº de entradas $x =$ prezo da entrada

a) $I(x) =$ prezo · nº de entradas $= x(2800-80x) = 2800x - 80x^2$

$I'(x) = 2800 - 160x = 0$

$x = \dfrac{2800}{160} = 17'50 €$

vemos o signo da derivada

$\underline{\text{creciente}\ \ _{17'50}\ \ \text{Decreciente}}$ $\Rightarrow$ Punto máximo $P(17'50,\ 24500)$

$f'(0)>0$ $\quad I(17'50) = 2800\cdot 17'50 - 80\cdot 17'50^2 = 2$

$f'(20)<0$

$\underline{\text{Solución}}$: A entrada debe valer 17'50 €

11.- $f(x) = x^3 - 3x + 2$

a) • Co eixe de abscisas ($y=0$)

$x^3 - 3x + 2 = 0$

$$\begin{array}{r|rrrr} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \lfloor 0 \\ 1 & & 1 & 2 & \\ \hline & 1 & 2 & \lfloor 0 & \\ -2 & & -2 & & \\ \hline & 1 & \lfloor 0 & & \end{array}$$

$P(1,0)$

$P(-2,0)$

• Co eixe de ordenadas (x

$y = 0^3 - 3\cdot 0 + 2 = 2$

$P(0,2)$

b) $f'(x) = 3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

$\underline{\text{creciente}\ _{-1}\ \text{decreciente}\ _{1}\ \text{creciente}}$

$f'(-2)>0$ $\quad f'(2)>0$

$f'(0)<0$

Creciente en $(-\infty,-1)\cup(1,\infty)$

Decreciente en $(-1,1)$

c) • Punto máximo $P(-1,4)$

$f(-1) = (-1)^3 - 3\cdot(-1) + 2 = -1 + 3 + 2 = 4$

• Punto mínimo $P(1,0)$

$f(1) = 1^3 - 3\cdot 1 + 2 = 0$

d) $f''(x) = 6x = 0$

$x = 0$

$\underset{0}{\text{cóncava} \mid \text{convexa}}$

$f''(-1) < 0$

$f''(1) > 0$

• cóncava en $(-\infty, 0)$

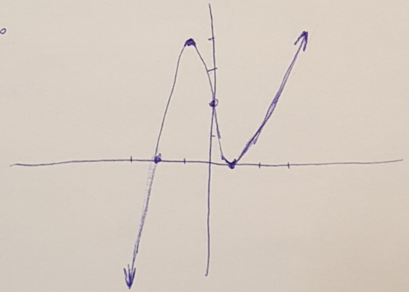
• Convexa en $(0, \infty)$

e) Punto de inflexión $P(0,2)$

$f(0) = 2$

f) $\lim\limits_{x \to \infty} f(x) = \infty$

$\lim\limits_{x \to -\infty} f(x) = -\infty$



x	y
-3	-16
3	20