# 6 NÚMEROS COMPLEXOS

## Páxina 147

## REFLEXIONA E RESOLVE

### Extraer fóra da raíz

■ **Saca fóra da raíz:**

**a)** $\sqrt{-16}$ **b)** $\sqrt{-100}$

a) $\sqrt{-16} = \sqrt{-1 \cdot 16} = 4\sqrt{-1}$

b) $\sqrt{-100} = 10\sqrt{-1}$

### Potencias de $\sqrt{-1}$

■ **Calcula as sucesivas potencias de $\sqrt{-1}$:**

**a)** $(\sqrt{-1})^3 = (\sqrt{-1})^2(\sqrt{-1}) = \ldots$ **b)** $(\sqrt{-1})^4$ **c)** $(\sqrt{-1})^5$

a) $(\sqrt{-1})^3 = (\sqrt{-1})^2(\sqrt{-1}) = (-1) \cdot \sqrt{-1} = -\sqrt{-1}$

b) $(\sqrt{-1})^4 = (\sqrt{-1})^2(\sqrt{-1})^2 = (-1) \cdot (-1) = 1$

c) $(\sqrt{-1})^5 = (\sqrt{-1})^4 \cdot \sqrt{-1} = 1 \cdot \sqrt{-1} = \sqrt{-1}$

### Como se manexa $k \cdot \sqrt{-1}$?

■ **Simplifica.**

**a)** $-2\sqrt{-1} + 11\sqrt{-1} - 8\sqrt{-1} - \sqrt{-1}$

**b)** $5\sqrt{-1} + 2\sqrt{-1} - 10\sqrt{-1} + 3\sqrt{-1}$

**c)** $8\sqrt{-1} + \frac{2}{5}\sqrt{-1} - \frac{3}{10}\sqrt{-1} - \frac{1}{2}\sqrt{-1}$

a) $-2\sqrt{-1} + 11\sqrt{-1} - 8\sqrt{-1} - \sqrt{-1} = 0 \cdot \sqrt{-1} = 0$

b) $5\sqrt{-1} + 2\sqrt{-1} - 10\sqrt{-1} + 3\sqrt{-1} = 0$

c) $8\sqrt{-1} + \frac{2}{5}\sqrt{-1} - \frac{3}{10}\sqrt{-1} - \frac{1}{2}\sqrt{-1} = \left(\frac{80}{10} + \frac{4}{10} - \frac{3}{10} - \frac{5}{10}\right)\sqrt{-1} = \frac{38}{5}\sqrt{-1}$

## Expresións do tipo $a + b \cdot \sqrt{-1}$

■ **Simplifica as seguintes sumas:**

**a)** $\left(-3 + 5\sqrt{-1}\right) + \left(2 - 4\sqrt{-1}\right) - \left(6\sqrt{-1}\right)$

**b)** $(-5)\left(5 + \sqrt{-1}\right) - 2\left(1 - 6\sqrt{-1}\right)$

a) $\left(-3 + 5\sqrt{-1}\right) + \left(2 - 4\sqrt{-1}\right) - \left(6\sqrt{-1}\right) = -1 - 5\sqrt{-1}$

b) $(-5)\left(5 + \sqrt{-1}\right) - 2\left(1 - 6\sqrt{-1}\right) = -3 - \sqrt{-1}$

■ **Efectúa as seguintes operacións combinadas:**

**a)** $3\left(2 - 4\sqrt{-1}\right) - 6\left(4 + 7\sqrt{-1}\right)$

**b)** $8\left(5 - 3\sqrt{-1}\right) + 4\left(-3 + 2\sqrt{-1}\right)$

a) $3\left(2 - 4\sqrt{-1}\right) - 6\left(4 + 7\sqrt{-1}\right) = 6 - 12\sqrt{-1} - 24 - 42\sqrt{-1} = -18 - 54\sqrt{-1}$

b) $8\left(5 - 3\sqrt{-1}\right) + 4\left(-3 + 2\sqrt{-1}\right) = 40 - 24\sqrt{-1} - 12 + 8\sqrt{-1} = 28 - 16\sqrt{-1}$

## Multiplicacións

■ **Efectúa as seguintes multiplicacións:**

**a)** $\left(4 - 3\sqrt{-1}\right) \cdot \sqrt{-1}$

**b)** $\left(5 + 2\sqrt{-1}\right) \cdot 8\sqrt{-1}$

**c)** $\left(5 + 2\sqrt{-1}\right)\left(7 - 3\sqrt{-1}\right)$

**d)** $\left(5 + 2\sqrt{-1}\right)\left(5 - 2\sqrt{-1}\right)$

a) $\left(4 - 3\sqrt{-1}\right) \cdot \sqrt{-1} = 4\sqrt{-1} - 3\left(\sqrt{-1}\right)^2 = 4\sqrt{-1} - 3\,(-1) = 3 + 4\sqrt{-1}$

b) $\left(5 + 2\sqrt{-1}\right) \cdot 8\sqrt{-1} = 40\sqrt{-1} + 16\left(\sqrt{-1}\right)^2 = -16 + 40\sqrt{-1}$

c) $\left(5 + 2\sqrt{-1}\right)\left(7 - 3\sqrt{-1}\right) = 35 - 15\sqrt{-1} + 14\sqrt{-1} - 6\left(\sqrt{-1}\right)^2 = 35 + 6 - \sqrt{1} = 41 - \sqrt{-1}$

d) $\left(5 + 2\sqrt{-1}\right)\left(5 - 2\sqrt{-1}\right) = 25 - 10\sqrt{-1} + 10\sqrt{-1} - 4\left(\sqrt{-1}\right)^2 = 25 + 4 = 29$

## Ecuacións de segundo grao

■ **Resolve:**

**a)** $x^2 + 10x + 29 = 0$

**b)** $x^2 + 9 = 0$

a) $x^2 + 10x + 29 = 0 \rightarrow x = \dfrac{-10 \pm \sqrt{100 - 116}}{2} = \dfrac{-10 \pm \sqrt{-16}}{2} = \dfrac{-10 \pm 4\sqrt{-1}}{2} =$

$= -5 \pm 2\sqrt{-1}$ ⟨ $x_1 = -5 + 2\sqrt{-1}$; $x_2 = -5 - 2\sqrt{-1}$

b) $x^2 + 9 = 0 \rightarrow x^2 = -9 \rightarrow x = \pm\sqrt{-9} = \pm 3\sqrt{-1}$ ⟨ $x_1 = 3\sqrt{-1}$; $x_2 = -3\sqrt{-1}$

## Páxina 149

**1. Representa graficamente os seguintes números complexos e di cales son reais, cales imaxinarios e, destes, cales son imaxinarios puros:**

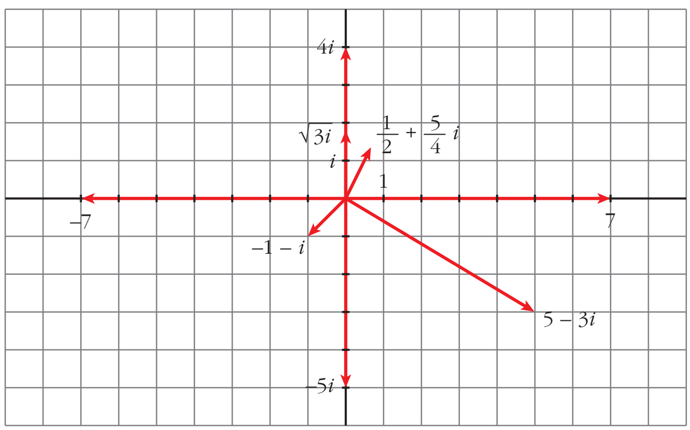
$$5 - 3i;\ \ \frac{1}{2} + \frac{5}{4}i;\ \ -5i;\ \ 7;\ \ \sqrt{3}\,i;\ \ 0;\ \ -1 - i;\ \ -7;\ \ 4i$$

- Reales: 7, 0 y –7

  Imaginarios: $5 - 3i$, $\frac{1}{2} + \frac{5}{4}i$, $-5i$, $\sqrt{3}\,i$, $-1 - i$, $4i$

  Imaginarios puros: $-5i$, $\sqrt{3}\,i$, $4i$

- Representación:




**2. Obtén as solucións das seguintes ecuacións e represéntaas:**

**a)** $z^2 + 4 = 0$

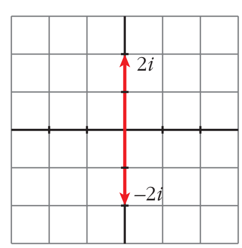
**b)** $z^2 + 6z + 10 = 0$

**c)** $3z^2 + 27 = 0$

**d)** $3z^2 - 27 = 0$

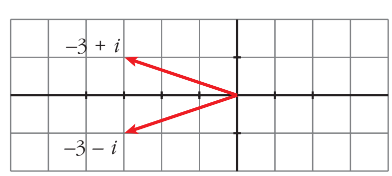
a) $z = \dfrac{\pm\sqrt{-16}}{2} = \dfrac{\pm 4i}{2} = \pm 2i$

$z_1 = 2i$, $z_2 = -2i$



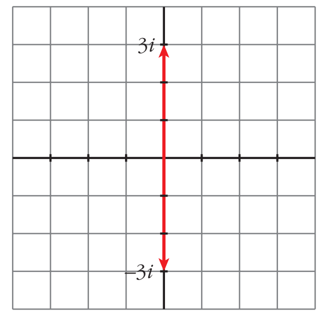

b) $z = \dfrac{-6 \pm \sqrt{36 - 40}}{2} = \dfrac{-6 \pm \sqrt{-4}}{2} =$

$= \dfrac{-6 \pm 2i}{2} = -3 \pm i$; $z_1 = -3 - i$, $z_2 = -3 + i$



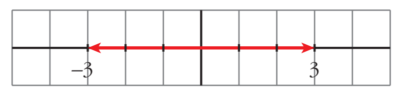
c) $z^2 = -9 \;\rightarrow\; z = \pm\sqrt{-9} = \pm 3i$

$z_1 = -3i, \;\; z_2 = 3i$




d) $z^2 = 9 \;\rightarrow\; z = \pm 3$

$z_1 = -3, \;\; z_2 = 3$



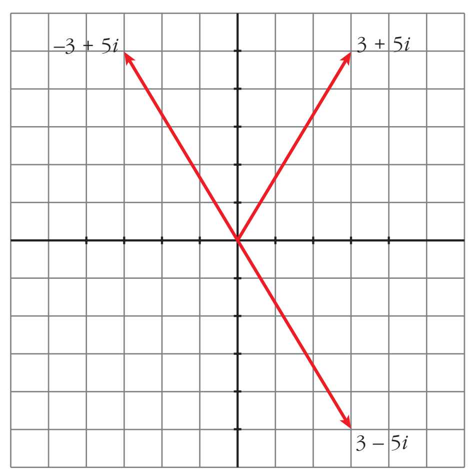

**3.** **Representa graficamente o oposto e mais o conxugado de:**

**a)** $\mathbf{3 - 5}\boldsymbol{i}$ **b)** $\mathbf{5 + 2}\boldsymbol{i}$ **c)** $\mathbf{-1 - 2}\boldsymbol{i}$ **d)** $\mathbf{-2 + 3}\boldsymbol{i}$

**e)** $\mathbf{5}$ **f)** $\mathbf{0}$ **g)** $\mathbf{2}\boldsymbol{i}$ **h)** $\mathbf{-5}\boldsymbol{i}$

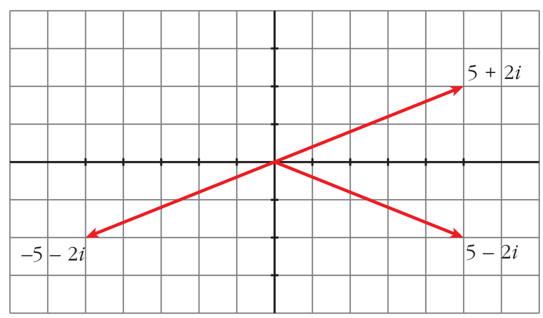
a) Opuesto: $-3 + 5i$

Conjugado: $3 + 5i$



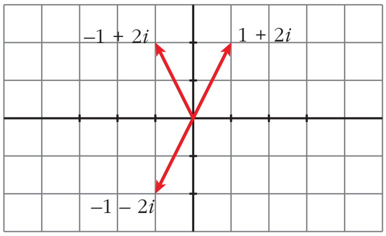

b) Opuesto: $-5 - 2i$

Conjugado: $5 - 2i$



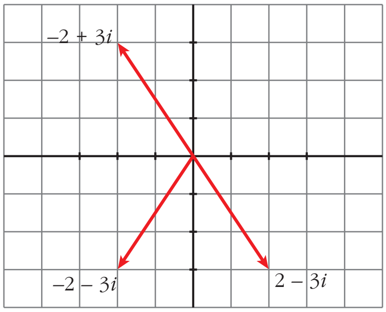
c) Opuesto: $1 + 2i$

Conjugado: $-1 + 2i$



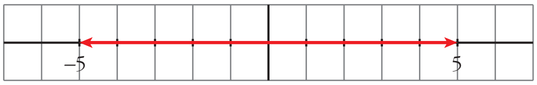

d) Opuesto: $2 - 3i$

Conjugado: $-2 - 3i$



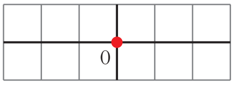

e) Opuesto: $-5$

Conjugado: $5$



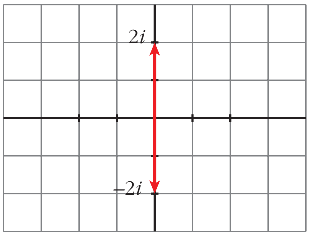

f) Opuesto: $0$

Conjugado: $0$



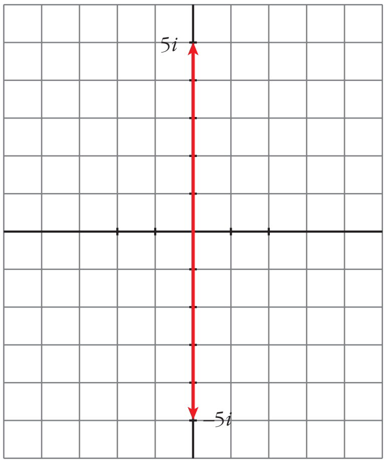

g) Opuesto: $-2i$

Conjugado: $-2i$




h) Opuesto: $5i$

Conjugado: $5i$



**4.** **Sabemos que $i^2 = -1$. Calcula $i^3$, $i^4$, $i^5$, $i^6$, $i^{20}$, $i^{21}$, $i^{22}$, $i^{23}$. Dá un criterio para simplificar potencias de $i$ de expoñente natural.**

$i^3 = -i$ $\quad i^4 = 1$ $\quad i^5 = i$ $\quad i^6 = -1$

$i^{20} = 1$ $\quad i^{21} = i$ $\quad i^{22} = -1$ $\quad i^{23} = -i$

CRITERIO: Dividimos el exponente entre 4 y lo escribimos como sigue:

$$i^n = i^{4c+r} = i^{4c} \cdot i^r = (i^4)^c \cdot i^r = 1^c \cdot i^r = 1 \cdot i^r = i^r$$

Por tanto, $i^n = i^r$, donde $r$ es el resto de dividir $n$ entre 4.

## Páxina 151

**1.** **Efectúa as seguintes operacións e simplifica o resultado:**

**a)** $(6 - 5i) + (2 - i) - 2(-5 + 6i)$

**b)** $(2 - 3i) - (5 + 4i) + \frac{1}{2}(6 - 4i)$

**c)** $(3 + 2i)(4 - 2i)$

**d)** $(2 + 3i)(5 - 6i)$

**e)** $(-i + 1)(3 - 2i)(1 + 3i)$

**f)** $\frac{2 + 4i}{4 - 2i}$ **g)** $\frac{1 - 4i}{3 + i}$ **h)** $\frac{4 + 4i}{-3 + 5i}$

**i)** $\frac{5 + i}{-2 - i}$ **l)** $\frac{1 + 5i}{3 + 4i}$ **m)** $\frac{4 - 2i}{i}$

**n)** $6 - 3\left(5 + \frac{2}{5}i\right)$ **ñ)** $\frac{(-3i)^2(1 - 2i)}{2 + 2i}$

a) $(6 - 5i) + (2 - i) - 2(-5 + 6i) = 6 - 5i + 2 - i + 10 - 12i = 18 - 18i$

b) $(2 - 3i) - (5 + 4i) + \frac{1}{2}(6 - 4i) = 2 - 3i - 5 - 4i + 3 - 2i = -9i$

c) $(3 + 2i)(4 - 2i) = 12 - 6i + 8i - 4i^2 = 12 + 2i + 4 = 16 + 2i$

d) $(2 + 3i)(5 - 6i) = 10 - 12i + 15i - 18i^2 = 10 + 3i + 18 = 28 + 3i$

e) $(-i + 1)(3 - 2i)(1 + 3i) = (-3i + 2i^2 + 3 - 2i)(1 + 3i) = (3 - 2 - 5i)(1 + 3i) =$

$= (1 - 5i)(1 + 3i) = 1 + 3i - 5i - 15i^2 = 1 + 15 - 2i = 16 - 2i$

f) $\frac{2 + 4i}{4 - 2i} = \frac{(2 + 4i)(4 + 2i)}{(4 - 2i)(4 + 2i)} = \frac{8 + 4i + 16i + 8i^2}{16 - 4i^2} = \frac{20i}{16 + 4} = \frac{20i}{20} = i$

g) $\frac{1 - 4i}{3 + i} = \frac{(1 - 4i)(3 - i)}{(3 + i)(3 - i)} = \frac{3 - i - 12i + 4i^2}{9 - i^2} = \frac{3 - 13i - 4}{9 + 1} = \frac{-1 - 13i}{10} =$

$= \frac{-1}{10} - \frac{13}{10}i$

h) $\frac{4+4i}{-3+5i} = \frac{(4+4i)(-3-5i)}{(-3+5i)(-3-5i)} = \frac{-12-20i-12i-20i^2}{9-25i^2} = \frac{-12-32i+20}{9+25} =$

$= \frac{8-32i}{34} = \frac{8}{34} - \frac{32}{34}i = \frac{4}{17} - \frac{16}{17}i$

i) $\frac{5+i}{-2-i} = \frac{(5+i)(-2+i)}{(-2-i)(-2+i)} = \frac{-10+5i-2i+i^2}{4+1} = \frac{-10+3i-1}{5} = \frac{-11+3i}{5} =$

$= \frac{-11}{5} + \frac{3}{5}i$

l) $\frac{1+5i}{3+4i} = \frac{(1+5i)(3-4i)}{(3+4i)(3-4i)} = \frac{3-4i+15i-20i^2}{9-16i^2} = \frac{3+11i+20}{9+16} =$

$= \frac{23+11i}{25} = \frac{23}{25} + \frac{11}{25}i$

m) $\frac{4-2i}{i} = \frac{(4-2i)(-i)}{i(-i)} = \frac{-4i+2i^2}{1} = -4i-2 = -2-4i$

n) $6 - 3\left(5 + \frac{2}{5}i\right) = 6 - 15 + \frac{6}{5}i = -9 + \frac{6}{5}i$

ñ) $\frac{(-3i)^2(1-2i)}{(2+2i)} = \frac{9i^2(1-2i)}{(2+2i)} = \frac{-9(1-2i)}{(2+2i)} = \frac{-9+18i}{(2+2i)} =$

$= \frac{(-9+18i)(2-2i)}{(2+2i)(2-2i)} = \frac{-18+18i+36i-36i^2}{4-4i^2} = \frac{-18+54i+36}{4+4} =$

$= \frac{18+54i}{8} = \frac{18}{8} + \frac{54}{8}i = \frac{9}{4} + \frac{27}{4}i$

**2. Obtén polinomios cuxas raíces sexan:**

**a)** $2+\sqrt{3}i$ **e** $2-\sqrt{3}i$ **b)** $-3i$ **e** $3i$ **c)** $1+2i$ **e** $3-4i$

**(Observa que só cando as dúas raíces son conxugadas, o polinomio ten coeficientes reais).**

a) $\left[x - \left(2 + \sqrt{3}i\right)\right]\left[x - \left(2 - \sqrt{3}i\right)\right] =$

$= \left[(x-2) - \sqrt{3}i\right]\left[(x-2) + \sqrt{3}i\right] = (x-2)^2 - \left(\sqrt{3}i\right)^2 =$

$= x^2 - 4x + 4 - 3i^2 = x^2 - 4x + 4 + 3 = x^2 - 4x + 7$

b) $[x - (-3i)][x - 3i] = [x + 3i][x - 3i] = x^2 - 9i^2 = x^2 + 9$

c) $[x - (1+2i)][x - (3-4i)] = [(x-1) - 2i][(x-3) + 4i] =$

$= (x-1)(x-3) + 4(x-1)i - 2(x-3)i - 8i^2 =$

$= x^2 - 4x + 3 + (4x - 4 - 2x + 6)i + 8 = x^2 - 4x + 11 + (2x+2)i =$

$= x^2 - 4x + 11 + 2ix + 2i = x^2 + (-4+2i)x + (11+2i)$

**3.** **Canto debe valer $x$, real, para que $(25 - xi)^2$ sexa imaxinario puro?**

$$(25 - xi)^2 = 625 + x^2 i^2 - 50xi = (625 - x^2) - 50xi$$

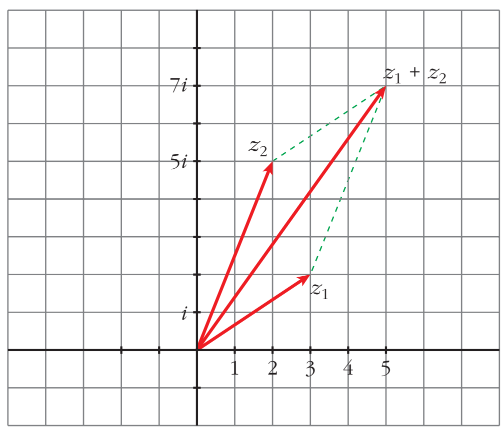
Para que sea imaginario puro:

$$625 - x^2 = 0 \rightarrow x^2 = 625 \rightarrow x = \pm\sqrt{625} = \pm 25$$

Hay dos soluciones: $x_1 = -25$, $x_2 = 25$

**4.** **Representa graficamente $z_1 = 3 + 2i$, $z_2 = 2 + 5i$, $z_1 + z_2$. Comproba que $z_1 + z_2$ é unha diagonal do paralelogramo de lados $z_1$ e $z_2$.**

$z_1 + z_2 = 5 + 7i$




## Páxina 153

**1.** **Escribe en forma polar os seguintes números complexos:**

**a)** $1 + \sqrt{3}i$ **b)** $\sqrt{3} + i$ **c)** $-1 + i$

**d)** $5 - 12i$ **e)** $3i$ **f)** $-5$

a) $1 + \sqrt{3}\, i = 2_{60°}$ b) $\sqrt{3} + i = 2_{30°}$ c) $-1 + i = \sqrt{2}_{135°}$

d) $5 - 12i = 13_{292°\, 37'}$ e) $3i = 3_{90°}$ f) $-5 = 5$

**2.** **Escribe en forma binómica os seguintes números complexos:**

**a)** $5_{(\pi/6)\ rad}$ **b)** $2_{135°}$ **c)** $2_{495°}$

**d)** $3_{240°}$ **e)** $5_{180°}$ **f)** $4_{90°}$

a) $5_{(\pi/6)} = 5\left(\cos \frac{\pi}{6} + i \operatorname{sen} \frac{\pi}{6}\right) = 5\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \frac{5\sqrt{3}}{2} + \frac{5}{2}i$

b) $2_{135°} = 2(\cos 135° + i \operatorname{sen} 135°) = 2\left(-\frac{\sqrt{2}}{2} + i\,\frac{\sqrt{2}}{2}\right) = -\sqrt{2} + \sqrt{2}i$

c) $2_{495°} = 2_{135°} = -\sqrt{2} + \sqrt{2}i$

d) $3_{240°} = 3(cos\ 240° + i\ sen\ 240°) = 3\left(-\frac{1}{2} - i\ \frac{\sqrt{3}}{2}\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$

e) $5_{180°} = -5$

f) $4_{90°} = 4i$

**3.** **Expresa en forma polar o oposto e o conxugado do número complexo $z = r_\alpha$.**

Opuesto: $-z = r_{180° + \alpha}$ Conjugado: $\bar{z} = r_{360° - \alpha}$

**4.** **Escribe en forma binómica e en forma polar o complexo:**

$$z = 8(cos\ 30° + i\ sen\ 30°)$$

$z = 8_{30°} = 8(cos\ 30° + i\ sen\ 30°) = 8\left(\frac{\sqrt{3}}{2} + i\ \frac{1}{2}\right) = \frac{8\sqrt{3}}{2} + \frac{8}{2}i = 4\sqrt{3} + 4i$

**5.** **Sexan os números complexos $z_1 = 4_{60°}$ e $z_2 = 3_{210°}$.**

**a) Expresa $z_1$ e $z_2$ en forma binómica.**

**b) Calcula $z_1 \cdot z_2$ e $z_2/z_1$, e pasa os resultados a forma polar.**

**c) Compara os módulos e os argumentos de $z_1 \cdot z_2$ e $z_2/z_1$ cos de $z_1$ e $z_2$ e intenta encontrar relacións entre eles.**

a) $z_1 = 4_{60°} = 4(cos\ 60° + i\ sen\ 60°) = 4\left(\frac{1}{2} + i\ \frac{\sqrt{3}}{2}\right) = 2 + 2\sqrt{3}i$

$z_2 = 3_{210°} = 3(cos\ 210° + i\ sen\ 210°) = 3\left(-\frac{\sqrt{3}}{2} - i\ \frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$

b) $z_1 \cdot z_2 = \left(2 + 2\sqrt{3}i\right)\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right) =$

$= -3\sqrt{3} - 3i - 9i - 3\sqrt{3}i^2 = -3\sqrt{3} - 12i + 3\sqrt{3} = -12i = 12_{270°}$

$$\frac{z_2}{z_1} = \frac{\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right)}{\left(2 + 2\sqrt{3}i\right)} = \frac{\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right)\left(2 - 2\sqrt{3}i\right)}{\left(2 + 2\sqrt{3}i\right)\left(2 - 2\sqrt{3}i\right)} =$$

$$= \frac{-3\sqrt{3} - 3i + 9i + 3\sqrt{3}i^2}{4 - 12i^2} = \frac{-3\sqrt{3} + 6i - 3\sqrt{3}}{4 + 12} = \frac{-6\sqrt{3} + 6i}{16} = \left(\frac{3}{4}\right)_{150°}$$

c) $z_1 \cdot z_2 = 4_{60°} \cdot 3_{210°} = (4 \cdot 3)_{60° + 210°} = 12_{270°}$

$$\frac{z_2}{z_1} = \frac{3_{210°}}{4_{60°}} = \left(\frac{3}{4}\right)_{210° - 60°} = \left(\frac{3}{4}\right)_{150°}$$

## Páxina 155

**1. Efectúa estas operacións e dá o resultado en forma polar e en forma binómica:**

**a)** $1_{150°} \cdot 5_{30°}$ **b)** $6_{45°} : 3_{15°}$ **c)** $2_{10°} \cdot 1_{40°} \cdot 3_{70°}$

**d)** $5_{(2\pi/3)rad} : 1_{60°}$ **e)** $(1 - \sqrt{3}\,i)^5$ **f)** $(3 + 2i) + (-3 + 2i)$

a) $1_{150°} \cdot 5_{30°} = 5_{180°} = -5$

b) $6_{45°} : 3_{15°} = 2_{30°} = 2(\cos 30° + i \operatorname{sen} 30°) = 2\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \sqrt{3} + i$

c) $2_{10°} \cdot 1_{40°} \cdot 3_{70°} = 6_{120°} = 6(\cos 120° + i \operatorname{sen} 120°) = 6\left(-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right) = -3 + 3\sqrt{3}\,i$

d) $5_{(2\pi/3)rad} : 1_{60°} = 5_{120°} : 1_{60°} = 5_{60°} = 5(\cos 60° + i \operatorname{sen} 60°) =$

$$= 5\left(\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right) = \frac{5}{2} + \frac{5\sqrt{3}}{2}\,i$$

e) $(1 - \sqrt{3}\,i)^5 = (2_{300°})^5 = 32_{1\,500°} = 32_{60°} = 32(\cos 60° + i \operatorname{sen} 60°) =$

$$= 32\left(\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right) = 16 + 16\sqrt{3}\,i$$

f) $4i = 4_{90°}$

**2. Compara os resultados en cada caso:**

**a)** $(2_{30°})^3$, $(2_{150°})^3$, $(2_{270°})^3$

**b)** $(2_{60°})^4$, $(2_{150°})^4$, $(2_{270°})^4$, $(2_{330°})^4$

a) $(2_{30°})^3 = 2^3_{3 \cdot 30°} = 8_{90°}$

$(2_{150°})^3 = 2^3_{3 \cdot 150°} = 8_{450°} = 8_{90°}$

$(2_{270°})^3 = 8_{3 \cdot 270°} = 8_{810°} = 8_{90°}$

b) $(2_{60°})^4 = 2^4_{4 \cdot 60°} = 16_{240°}$

$(2_{150°})^4 = 16_{600°} = 16_{240°}$

$(2_{270°})^4 = 16_{1\,080°} = 16_{0°}$

$(2_{330°})^4 = 16_{1\,320°} = 16_{240°}$

**3. Dados os complexos** $z = 5_{45°}$, $w = 2_{15°}$, $t = 4i$, **obtén en forma polar:**

**a)** $z \cdot t$, **b)** $\frac{z}{w^2}$ **c)** $\frac{z^3}{w \cdot t^2}$ **d)** $\frac{z \cdot w^3}{t}$

$z = 5_{45°}$ $w = 2_{15°}$ $t = 4i = 4_{90°}$

a) $z \cdot w = 10_{60°}$

b) $\dfrac{z}{w^2} = \dfrac{z}{4_{30°}} = \dfrac{5_{45°}}{4_{30°}} = \left(\dfrac{5}{4}\right)_{15°}$

c) $\dfrac{z^3}{w \cdot t^2} = \dfrac{125_{135°}}{2_{15°} \cdot 16_{180°}} = \left(\dfrac{125}{32}\right)_{-60°} = \left(\dfrac{125}{32}\right)_{300°}$

d) $\dfrac{z \cdot w^3}{t} = \dfrac{5_{45°} \cdot 8_{45°}}{4_{90°}} = 10_{0°} = 10$

**4.** **Expresa *cos* 3α e *sen* 3α en función de *sen* α e *cos* α utilizando a fórmula de Moivre. Ten en conta que:**

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$(1_\alpha)^3 = 1\,(\cos\alpha + i \,\text{sen}\,\alpha)^3 =$

$= \cos^3\alpha + i\, 3\cos^2\alpha\,\text{sen}\,\alpha + 3i^2\cos\alpha\,\text{sen}^2\alpha + i^3\,\text{sen}^3\alpha =$

$= \cos^3\alpha + 3\cos^2\alpha\,\text{sen}\,\alpha\, i - 3\cos\alpha\,\text{sen}^2\alpha - i\,\text{sen}^3\alpha =$

$= (\cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha) + (3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha)\,i$

Por otra parte: $(1_\alpha)^3 = 1_{3\alpha} = \cos 3\alpha + i\,\text{sen}\,3\alpha$

Por tanto: $\cos 3\alpha = \cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha$

$\text{sen}\,3\alpha = 3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha$

## Páxina 157

**1.** **Calcula as seis raíces sextas de 1. Represéntaas e exprésaas en forma binómica.**

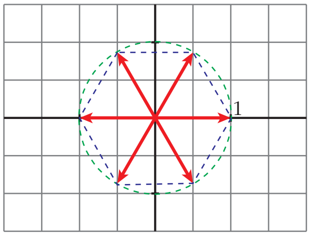
$\sqrt[6]{1} = \sqrt[6]{1_{0°}} = 1_{(360° \cdot k)/6} = 1_{60° \cdot k}; \quad k = 0, 1, 2, 3, 4, 5$

Las seis raíces son:

$1_{0°} = 1 \qquad 1_{60°} = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i \qquad 1_{120°} = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

$1_{180°} = -1 \qquad 1_{240°} = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i \qquad 1_{300°} = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

Representación:



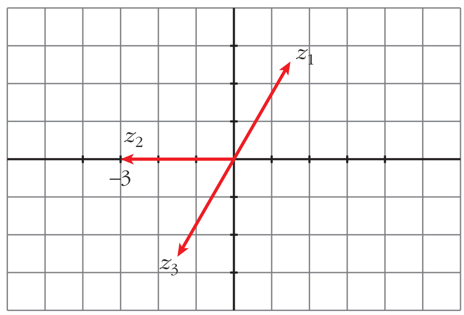
**2.** **Resolve a ecuación $z^3 + 27 = 0$. Representa as súas solucións.**

$$z^3 + 27 = 0 \rightarrow z = \sqrt[3]{-27} = \sqrt[3]{27_{180°}} = 3_{(180° + 360°\, n)/3} = 3_{60° + 120°\, n};\quad n = 0, 1, 2$$

$$z_1 = 3_{60°} = 3(\cos 60° + i \operatorname{sen} 60°) = 3\left(\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}\,i$$

$$z_2 = 3_{180°} = -3$$

$$z_3 = 3_{240°} = 3(\cos 240° + i \operatorname{sen} 240°) = 3\left(-\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}\,i$$




**3.** **Calcula:**

**a)** $\sqrt[3]{-i}$ **b)** $\sqrt[4]{-8 + 8\sqrt{3}i}$ **c)** $\sqrt{-25}$ **d)** $\sqrt{\dfrac{-2 + 2i}{1 + \sqrt{3}i}}$

a) $\sqrt[3]{-i} = \sqrt[3]{1_{270°}} = 1_{(270° + 360°\, k)/3};\quad k = 0, 1, 2$

Las tres raíces son:

$1_{90°} = i \qquad 1_{210°} = -\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\,i \qquad 1_{330°} = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\,i$

b) $\sqrt[4]{-8 + 8\sqrt{3}i} = \sqrt[4]{16_{120°}} = 2_{(120° + 360°\, k)/4} = 2_{30° + 90°\, k};\quad k = 0, 1, 2, 3$

Las cuatro raíces son:

$$2_{30°} = 2\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \sqrt{3} + i$$

$$2_{120°} = 2\left(-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right) = -1 + \sqrt{3}\,i$$

$$2_{210°} = 2\left(-\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right) = -1 - \sqrt{3}\,i$$

$$2_{300°} = 2\left(\frac{\sqrt{3}}{2} - i\,\frac{1}{2}\right) = \sqrt{3} - i$$

c) $\sqrt{-25} = \sqrt{25_{180°}} = 5_{(180° + 360°\,k)/2} = 5_{90° + 180°\,k}$; $k = 0, 1$

Las dos raíces son: $5_{90°} = 5i$; $5_{270°} = -5i$

d) $\sqrt[3]{\dfrac{-2 + 2i}{1 + \sqrt{3}\,i}} = \sqrt[3]{\dfrac{\sqrt{8}_{135°}}{2_{60°}}} = \sqrt[3]{\sqrt{2}_{75°}} = \sqrt[6]{2}_{(75° + 360°\,k)/3} = \sqrt[6]{2}_{25° + 120°\,k}$; $k = 0, 1, 2$

Las tres raíces son: $\sqrt[6]{2}_{25°}$; $\sqrt[6]{2}_{145°}$; $\sqrt[6]{2}_{265°}$

**4. Resolve as ecuacións:**

**a) $z^4 + 1 = 0$**

**b) $z^6 + 64 = 0$**

a) $z^4 + 1 = 0 \rightarrow z = \sqrt[4]{-1} = \sqrt[4]{1_{180°}} = 1_{(180° + 360°\,k)/2} = 1_{45° + 90°\,k}$; $k = 0, 1, 2, 3$

Las cuatro raíces son:

$1_{45°} = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i$; $1_{135°} = -\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i$; $1_{225°} = -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i$; $1_{315°} = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i$

b) $z^6 + 64 = 0 \rightarrow z = \sqrt[6]{-64} = \sqrt[6]{64_{180°}} = 2_{(180° + 360°\,k)/6} = 2_{30° + 60°\,k}$; $k = 0, 1, 2, 3, 4, 5$

Las seis raíces son:

$2_{30°} = 2\left(\dfrac{\sqrt{3}}{2} + i\,\dfrac{1}{2}\right) = \sqrt{3} + 1$

$2_{90°} = 2i$

$2_{150°} = 2\left(-\dfrac{\sqrt{3}}{2} + i\,\dfrac{1}{2}\right) = -\sqrt{3} + i$

$2_{210°} = 2\left(-\dfrac{\sqrt{3}}{2} - i\,\dfrac{1}{2}\right) = -\sqrt{3} - i$

$2_{270°} = -2i$

$2_{330°} = 2\left(\dfrac{\sqrt{3}}{2} - i\,\dfrac{1}{2}\right) = \sqrt{3} - i$

**5. Comproba que se $z$ e $w$ son dúas raíces sextas de 1, daquela tamén o son os resultados das seguintes operacións:**

$$z \cdot w, \; z/w, \; z^2, \; z^3$$

$z$ y $w$ raíces sextas de 1 $\rightarrow z^6 = 1$, $w^6 = 1$

$(z \cdot w)^6 = z^6 \cdot w^6 = 1 \cdot 1 = 1 \rightarrow z \cdot w$ es raíz sexta de 1.

$\left(\dfrac{z}{w}\right)^6 = \dfrac{z^6}{w^6} = \dfrac{1}{1} = 1 \rightarrow \dfrac{z}{w}$ es raíz sexta de 1.

$z^2 = (z^2)^6 = z^{12} = (z^4)^3 = 1^3 = 1 \rightarrow z^2$ es raíz sexta de 1.

$z^3 = (z^3)^6 = z^{18} = z^{16} \cdot z^2 = (z^4)^4 \cdot z^2 = 1^4 \cdot 1^2 = 1 \cdot 1 = 1 \rightarrow z^3$ es raíz sexta de 1.

**6.** **O número $4 + 3i$ é a raíz cuarta dun certo número complexo, $z$. Calcula as outras tres raíces cuartas de $z$.**

$4 + 3i = 5_{36^\circ\ 52'}$

Las otras tres raíces cuartas de $z$ serán:

$5_{36^\circ\ 52' + 90^\circ} = 5_{126^\circ\ 52'} = -3 + 4i$

$5_{36^\circ\ 52' + 180^\circ} = 5_{216^\circ\ 52'} = -4 - 3i$

$5_{36^\circ\ 52' + 270^\circ} = 5_{306^\circ\ 52'} = 3 - 4i$

**7.** **Calcula as seguintes raíces e representa graficamente as súas solucións:**

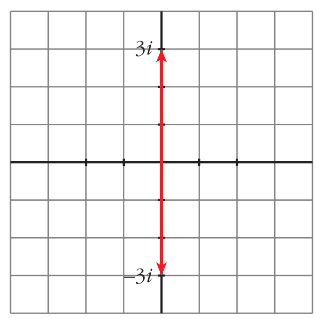
**a)** $\sqrt[3]{-9}$ **b)** $\sqrt[3]{-27}$ **c)** $\sqrt[3]{2 - 2i}$

**d)** $\sqrt[3]{\dfrac{1 - i}{1 + i}}$ **e)** $\sqrt[5]{\dfrac{32}{i}}$ **f)** $\sqrt[3]{8i}$

a) $\sqrt{-9} = \sqrt{9_{180^\circ}} = 3_{(180^\circ + 360^\circ k)/2} = 3_{90^\circ + 180^\circ k}$; $k = 0, 1$

Las dos raíces son:

$3_{90^\circ} = 3i$; $3_{270^\circ} = -3i$



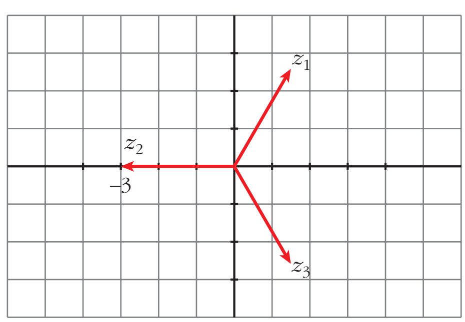

b) $\sqrt[3]{-27} = \sqrt[3]{27_{180^\circ}} = 3_{(180^\circ + 360^\circ k)/3} = 3_{60^\circ + 120^\circ k}$; $k = 0, 1, 2$

Las tres raíces son:

$z_1 = 3_{60^\circ} = 3(\cos 60^\circ + i \operatorname{sen} 60^\circ) = 3\left(\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right) = \dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}i$

$z_2 = 3_{180^\circ} = -3$

$z_3 = 3_{300^\circ} = 3(\cos 300^\circ + i \operatorname{sen} 300^\circ) = 3\left(\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}\right) = \dfrac{3}{2} - \dfrac{3\sqrt{3}}{2}i$



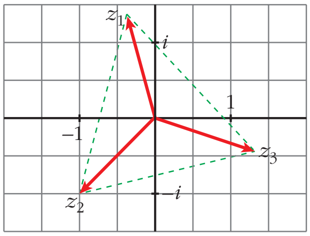
c) $\sqrt[3]{2-2i} = \sqrt[3]{\sqrt{8}_{315°}} = \sqrt{2}_{(315° + 360°\,k)/3} = \sqrt{2}_{105° + 120°\,k}$; $k = 0, 1, 2$

Las tres raíces son:

$z_1 = \sqrt{2}_{105°} = -0,37 + 1,37i$

$z_2 = \sqrt{2}_{225°} = \sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = -1 - i$

$z_3 = \sqrt{2}_{345°} = 1,37 - 0,37i$



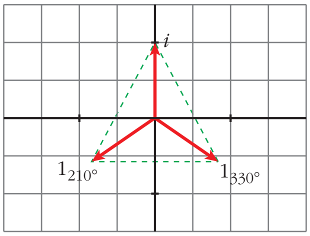

d) $\sqrt[3]{\frac{1-i}{1+i}} = \sqrt[3]{\frac{\sqrt{2}_{315°}}{\sqrt{2}_{45°}}} = \sqrt[3]{1_{270°}} = 1_{(270° + 360°\,k)/3} = 1_{90° + 120°\,k}$; $k = 0, 1, 2$

Las tres raíces son:

$1_{90°} = i$

$1_{210°} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

$1_{330°} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$




e) $\sqrt[5]{-\frac{32}{i}} = \sqrt[5]{-\frac{32\,(-i)}{i\,(-i)}} = \sqrt[5]{32i} = \sqrt[5]{32_{90°}} = 2_{(90° + 360°\,k)/5} = 2_{18° + 72°\,k}$; $k = 0, 1, 2, 3, 4$

Las cinco raíces son:

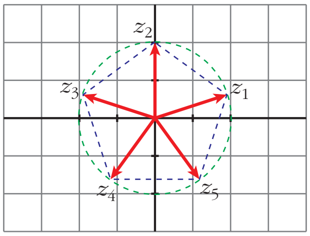
$z_1 = 2_{18°} = 1,9 + 0,6i$

$z_2 = 2_{90°} = 2i$

$z_3 = 2_{162°} = -1,9 + 0,6i$

$z_4 = 2_{234°} = -1,2 - 1,6i$

$z_5 = 2_{306°} = 1,2 - 1,6i$



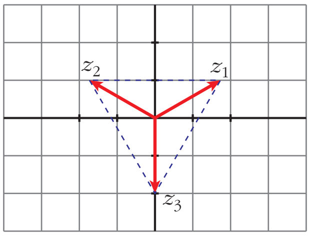

f) $\sqrt[3]{8i} = \sqrt[3]{8_{90°}} = 2_{(90° + 360°\,k)/3} = 2_{30° + 120°\,k}$; $k = 0, 1, 2$

Las tres raíces son:

$z_1 = 2_{30°}$

$z_2 = 2_{150°}$

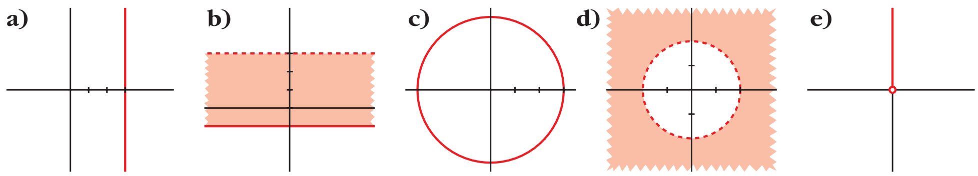
$z_3 = 2_{270°}$



## Páxina 158

# LINGUAXE MATEMÁTICA

**1. Pon a ecuación ou inecuación que caracteriza os seguintes recintos ou liñas:**



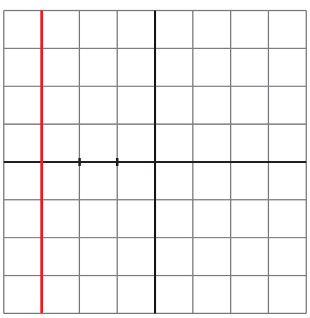

**Describe con palabras cada unha das familias ("son os números complexos cuxa parte real vale ...") e dá un representante de cada unha delas.**

a) $Re\ z = 3$  b) $-1 \leq Im\ z < 3$  c) $|z| = 3$  d) $|z| > 2$  e) $Arg\ z = 90°$

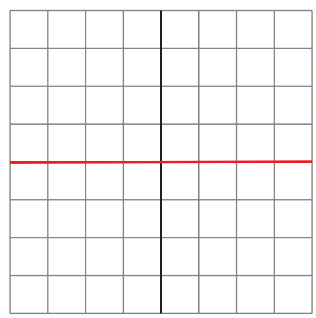
**2. Representa:**

**a)** $Re\ z = -3$  **b)** $Im\ z = 0$  **c)** $3 < Re\ z \leq 5$  **d)** $|z| \geq 4$  **e)** $Arg\ z = 180°$

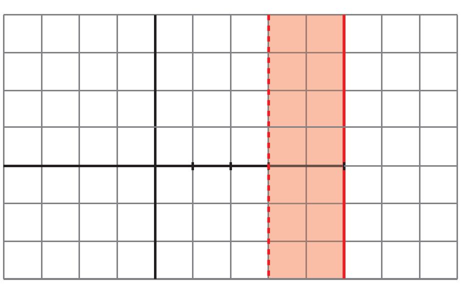
a)



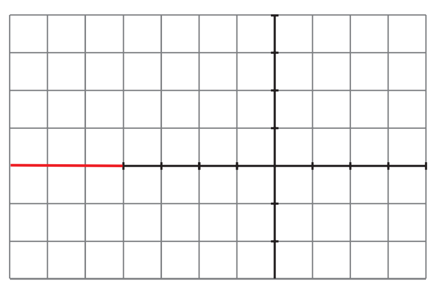
b)



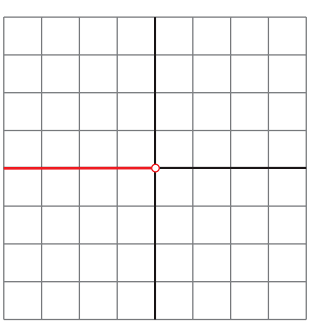
c)



d)



e)



## Páxina 162

# EXERCICIOS E PROBLEMAS PROPOSTOS

### PARA PRACTICAR

### Números complexos en forma binómica

**1** **Calcula:**

**a)** $(3+2i)(2-i)-(1-i)(2-3i)$ **b)** $3+2i(-1+i)-(5-4i)$

**c)** $-2i-(4-i)5i$ **d)** $(4-3i)(4+3i)-(4-3i)^2$

a) $(3+2i)(2-i)-(1-i)(2-3i) = 6-3i+4i-2i^2-2+3i+2i-3i^2 =$

$= 6-3i+4i+2-2+3i+2i+3 = 9+6i$

b) $3+2i(-1+i)-(5-4i) = 3-2i+2i^2-5+4i = 3-2i-2-5+4i = -4+2i$

c) $-2i-(4-i)5i = -2i-20i+5i^2 = -22i-5 = -5-22i$

d) $(4-3i)(4+3i)-(4-3i)^2 = 16-(3i)^2-16-9i^2+24i =$

$= 16+9-16+9+24i = 18+24i$

**2** **Calcula en forma binómica:**

**a)** $\dfrac{(3+3i)(4-2i)}{2-2i}$ **b)** $\dfrac{-2+3i}{(4+2i)(-1+i)}$

**c)** $\dfrac{2+5i}{3-2i}(1-i)$ **d)** $\dfrac{1+i}{2-i}+\dfrac{-3-2i}{1+3i}$

a) $\dfrac{(3+3i)(4-2i)}{2-2i} = \dfrac{12-6i+12i-6i^2}{2-2i} = \dfrac{18+6i}{2-2i} = \dfrac{(18+6i)(2+2i)}{(2-2i)(2+2i)} =$

$= \dfrac{36+36i+12i-12}{4+4} = \dfrac{24+48i}{8} = 3+6i$

b) $\dfrac{-2+3i}{(4+2i)(-1+i)} = \dfrac{-2+3i}{-4+4i-2i-2} = \dfrac{-2+3i}{-6+2i} = \dfrac{(-2+3i)(-6-2i)}{(-6+2i)(-6-2i)} =$

$= \dfrac{12+4i-18i+6}{36+4} = \dfrac{18-14i}{40} = \dfrac{9-7i}{20} = \dfrac{9}{20}-\dfrac{7}{20}i$

c) $\dfrac{2+5i}{3-2i}(1-i) = \dfrac{2-2i+5i+5}{3-2i} = \dfrac{7+3i}{3-2i} = \dfrac{(7+3i)(3+2i)}{(3-2i)(3+2i)} =$

$= \dfrac{21+14i+9i-6}{9+4} = \dfrac{15+23i}{13} = \dfrac{15}{13}+\dfrac{23}{13}i$

d) $\dfrac{1+i}{2-i} + \dfrac{-3-2i}{1+3i} = \dfrac{(1+i)(2+i)}{(2-i)(2+i)} + \dfrac{(-3-2i)(1-3i)}{(1+3i)(1-3i)} =$

$= \dfrac{2+i+2i-1}{4+1} + \dfrac{-3+9i-2i-6}{1+9} = \dfrac{1+3i}{5} + \dfrac{-9+7i}{10} =$

$= \dfrac{2+6i-9+7i}{10} = \dfrac{-7+13i}{10} = \dfrac{-7}{10} + \dfrac{13}{10}i$

**3** **Dados os números complexos $z = 1 - 3i$, $w = -3 + 2i$, $t = -2i$, calcula:**

**a)** $zwt$ **b)** $zt - w(t + z)$ **c)** $\dfrac{w}{z}t$

**d)** $\dfrac{2z - 3t}{w}$ **e)** $\dfrac{3z + it}{3}w$ **f)** $\dfrac{z^2 - wt^2}{2}$

$z = 1 - 3i$; $w = -3 + 2i$; $t = -2i$

a) $zwt = (1-3i)(-3+2i)(-2i) = (-3+2i+9i-6i^2)(-2i) =$

$= (3+11i)(-2i) = -6i - 22i^2 = 22 - 6i$

b) $zt - w(t+z) = (1-3i)(-2i) - (-3+2i)(-2i+1-3i) =$

$= (-2i+6i^2) - (-3+2i)(1-5i) = (-6-2i) - (-3+2i)(1-5i) =$

$= (-6-2i) - (-3+15i+2i-10i^2) = (-6-2i) - (7+17i) = -13 - 19i$

c) $\dfrac{w}{z}t = \dfrac{-3+2i}{1-3i}(-2i) = \dfrac{6i-4i^2}{1-3i} = \dfrac{(4+6i)(1+3i)}{1^2-(3i)^2} =$

$= \dfrac{4+12i+6i+18i^2}{1+9} = \dfrac{-14+18i}{10} = -\dfrac{7}{5} + \dfrac{9}{5}i$

d) $\dfrac{2z-3t}{w} = \dfrac{2(1-3i)-3(-2i)}{-3+2i} = \dfrac{2-6i+6i}{-3+2i} = \dfrac{2(-3-2i)}{(-3)^2-(2i)^2} =$

$= \dfrac{-6-4i}{9+4} = -\dfrac{6}{13} - \dfrac{4}{13}i$

e) $\dfrac{3z+it}{3}w = \dfrac{3(1-3i)+i(-2i)}{3}(-3+2i) = \dfrac{3-9i+2}{3}(-3+2i) =$

$= \left(\dfrac{5}{3} - 3i\right)(-3+2i) = -5 + \dfrac{10}{3}i + 9i - 6i^2 = 1 + \dfrac{37}{3}i$

f) $\dfrac{z^2-wt^2}{2} = \dfrac{(1-3i)^2-(-3+2i)(-2i)^2}{2} = \dfrac{1-6i+9i^2-(-3+2i)(-4)}{2} =$

$= \dfrac{-8-6i-12+8i}{2} = \dfrac{-20}{2} + \dfrac{2}{2}i = -10 + i$

**4** **Calcula:**

**a)** $i^{37}$ **b)** $i^{126}$ **c)** $i^{-7}$ **d)** $i^{64}$ **e)** $i^{-216}$

a) $i^{37} = i^1 = i$

b) $i^{126} = i^2 = -1$

c) $i^{-7} = \frac{1}{i^7} = \frac{1}{-i} = i$

d) $i^{64} = i^0 = 1$

e) $i^{-216} = \frac{1}{i^{216}} = \frac{1}{i^0} = \frac{1}{1} = 1$

**5** **Dado o número complexo** $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$**, proba que:**

**a)** $1 + z + z^2 = 0$

**b)** $\frac{1}{z} = z^2$

a) $z^2 = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 = \frac{1}{4} + \frac{3}{4}i^2 - \frac{\sqrt{3}}{2}i = \frac{1}{4} - \frac{3}{4} - \frac{\sqrt{3}}{2}i =$

$= -\frac{2}{4} - \frac{\sqrt{3}}{2}i = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$1 + z + z^2 = 1 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1 - \frac{1}{2} + \frac{\sqrt{3}}{2}i - \frac{1}{2} - \frac{\sqrt{3}}{2}i = 0$

b) $\frac{1}{z} = \frac{1}{-\frac{1}{2} + \frac{\sqrt{3}}{2}i} = \frac{1}{\frac{-1 + \sqrt{3}i}{2}} = \frac{2}{-1 + \sqrt{3}i} = \frac{2\left(-1 - \sqrt{3}i\right)}{(-1 + \sqrt{3}i)(-1 - \sqrt{3}i)} =$

$= \frac{2\left(-1 - \sqrt{3}i\right)}{1 + 3} = \frac{2\left(-1 - \sqrt{3}i\right)}{4} = \frac{-1 - \sqrt{3}i}{2} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$z^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ (lo habíamos calculado en a)

Por tanto: $\frac{1}{z} = z^2$

## Igualdade de números complexos

**6** **Calcula** ***m*** **e** ***n*** **para que se verifique a igualdade** $(2 + mi) + (n + 5i) = 7 - 2i$**.**

$(2 + mi) + (n + 5i) = 7 - 2i$

$(2 + n) + (m + 5)i = 7 - 2i \rightarrow \left.\begin{cases} 2 + n = 7 \\ m + 5 = -2 \end{cases}\right\} \begin{matrix} n = 5 \\ m = -7 \end{matrix}$

**7** **Determina $k$ para que o cociente $\dfrac{k+i}{1+i}$ sexa igual a $2 - i$.**

$$\frac{k+i}{1+i} = \frac{(k+i)(1-i)}{(1+i)(1-i)} = \frac{k - ki + i + 1}{1+1} = \frac{(k+1) + (1-k)i}{2} =$$

$$= \left(\frac{k+1}{2}\right) + \left(\frac{1-k}{2}\right) i = 2 - i \rightarrow \begin{cases} \dfrac{k+1}{2} = 2 \rightarrow k = 3 \\ \dfrac{1-k}{2} = -1 \rightarrow k = 3 \end{cases}$$

Por tanto, $k = 3$.

**8** **Calcula $a$ e $b$ de modo que se verifique:**

$$(a + bi)^2 = 3 + 4i$$

☛ ***Desenvolve o cadrado; iguala a parte real a 3, e a parte imaxinaria a 4.***

$(a + bi)^2 = 3 + 4i$

$a^2 + bi^2 + 2abi = 3 + 4i$

$a^2 - b^2 + 2abi = 3 + 4i \rightarrow \begin{cases} a^2 - b^2 = 3 \\ 2ab = 4 \end{cases}$

$b = \dfrac{4}{2a} = \dfrac{2}{a}$

$a^2 - \left(\dfrac{2}{a}\right)^2 = 3 \rightarrow a^2 - \dfrac{4}{a^2} = 3 \rightarrow a^4 - 4 = 3a^2 \rightarrow a^4 - 3a^2 - 4 = 0$

$a^2 = \dfrac{3 \pm \sqrt{9 + 16}}{2} = \dfrac{3 \pm 5}{2} \begin{cases} a^2 = 4 \rightarrow a = \pm 2 \\ a^2 = -1 \ \text{(no vale)} \end{cases}$

$a = -2 \rightarrow b = -1$

$a = 2 \rightarrow b = 1$

**9** **Dados os complexos $2 - ai$ e $3 - bi$, calcula $a$ e $b$ para que o produto sexa igual a $8 + 4i$.**

$(2 - ai)(3 - bi) = 8 + 4i$

$6 - 2bi - 3ai + abi^2 = 8 + 4i$

$6 - 2bi - 3ai - ab = 8 + 4i$

$(6 - ab) + (-2b - 3a)i = 8 + 4i$

$\begin{cases} 6 - ab = 8 \\ -2b - 3a = 4 \end{cases}$

$b = \dfrac{4 + 3a}{-2}$

$$6 - a\left(\frac{4 + 3a}{-2}\right) = 8 \;\rightarrow\; 6 + \frac{4a + 3a^2}{2} = 8$$

$$\frac{4a + 3a^2}{2} = 2 \;\rightarrow\; 4a + 3a^2 = 4 \;\rightarrow\; 3a^2 + 4a - 4 = 0$$

$$a = \frac{-4 \pm \sqrt{16 + 48}}{6} = \frac{-4 \pm 8}{6} \begin{cases} a = \dfrac{4}{6} = \dfrac{2}{3} \;\rightarrow\; b = -3 \\ a = \dfrac{-12}{6} = -2 \;\rightarrow\; b = 1 \end{cases}$$

**10** **Calcula o valor de $a$ e $b$ para que se verifique:**

$$a - 3i = \frac{2 + bi}{5 - 3i}$$

$a - 3i = \dfrac{2 + bi}{5 - 3i}$

$(a - 3i)\,(5 - 3i) = 2 + bi$

$5a - 3ai - 15i - 9 = 2 + bi$

$(5a - 9) + (-3a - 15)\,i = 2 + bi$

$$\left.\begin{array}{l} 5a - 9 = 2 \\ -3a - 15 = b \end{array}\right\} \begin{array}{l} a = 11/5 \\ b = -108/5 \end{array}$$

**11** **Calcula o valor de $b$ para que o produto $(3 - 6i)\,(4 + bi)$ sexa un número:**

**a) Imaxinario puro.** **b) Real.**

$$(3 - 6i)\,(4 + bi) = 12 + 3bi - 24i + 6b = (12 + 6b) + (3b - 24)\,i$$

a) $12 + 6b = 0 \;\rightarrow\; b = -2$

b) $3b - 24 = 0 \;\rightarrow\; b = 8$

**12** **Determina $a$ para que $(a - 2i)^2$ sexa un número imaxinario puro.**

$$(a - 2i)^2 = a^2 + 4i^2 - 4ai = (a^2 - 4) - 4ai$$

Para que sea imaginario puro, ha de ser:

$$a^2 - 4 = 0 \;\rightarrow\; a = \pm 2 \;\rightarrow\; a_1 = -2,\; a_2 = 2$$

**13** **Calcula $x$ para que o resultado do produto $(x + 2 + ix)\,(x - i)$ sexa un número real.**

$(x + 2 + ix)\,(x - i) = x^2 - xi + 2x - 2i + x^2 i - xi^2 =$

$$= x^2 - xi + 2x - 2i + ix^2 + x = (x^2 + 3x) + (x^2 - x - 2)i$$

Para que sea real, ha de ser:

$$x^2 - x - 2 = 0 \;\rightarrow\; x = \frac{1 \pm \sqrt{1 + 8}}{2} = \frac{1 \pm 3}{2} \begin{cases} x_1 = -1 \\ x_2 = 2 \end{cases}$$

## Números complexos en forma polar

**14** **Representa estes números complexos, os opostos e os conxugados. Exprésaos en forma polar.**

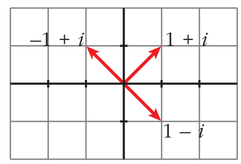
**a)** $1 - i$ **b)** $-1 + i$ **c)** $\sqrt{3} + i$ **d)** $-\sqrt{3} - i$

**e)** $-4$ **f)** $2i$ **g)** $-\frac{3}{4}i$ **h)** $2 + 2\sqrt{3}i$

a) $1 - i = \sqrt{2}_{315°}$

Opuesto: $-1 + i = \sqrt{2}_{135°}$

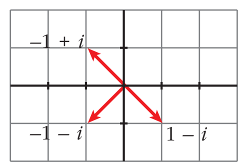
Conjugado: $1 + i = \sqrt{2}_{45°}$




b) $-1 + i = \sqrt{2}_{135°}$

Opuesto: $1 - i = \sqrt{2}_{315°}$

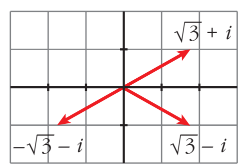
Conjugado: $-1 - i = \sqrt{2}_{225°}$




c) $\sqrt{3} + i = 2_{30°}$

Opuesto: $-\sqrt{3} - i = 2_{210°}$

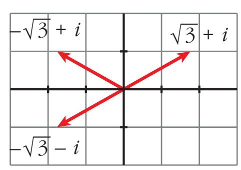
Conjugado: $\sqrt{3} - i = 2_{330°}$




d) $-\sqrt{3} - i = 2_{210°}$

Opuesto: $\sqrt{3} + i = 2_{30°}$

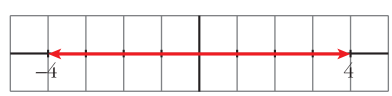
Conjugado: $-\sqrt{3} + i = 2_{150°}$




e) $-4 = 4_{180°}$

Opuesto: $4 = 4_{0°}$

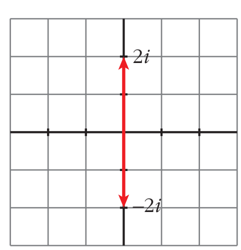
Conjugado: $-4 = 4_{180°}$




f) $2i = 2_{90°}$

Opuesto: $-2i = 2_{270°}$

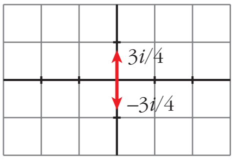
Conjugado: $-2i = 2_{270°}$



g) $-\frac{3}{4}i = \left(\frac{3}{4}\right)_{270°}$

Opuesto: $\frac{3}{4}i = \left(\frac{3}{4}\right)_{90°}$

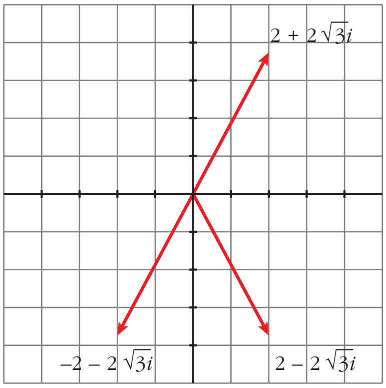
Conjugado: $\frac{3}{4}i = \left(\frac{3}{4}\right)_{90°}$




h) $2 + 2\sqrt{3}\,i = \sqrt{14}_{\,60°}$

Opuesto: $-2 - 2\sqrt{3}\,i = \sqrt{14}_{\,240°}$

Conjugado: $2 - 2\sqrt{3}\,i = \sqrt{14}_{\,300°}$




**15** **Escribe en forma binómica os seguintes números complexos:**

**a)** $2_{45°}$ **b)** $3_{(\pi/6)}$ **c)** $\sqrt{2}_{180°}$ **d)** $17_{0°}$

**e)** $1_{(\pi/2)}$ **f)** $5_{270°}$ **g)** $1_{150°}$ **h)** $4_{100°}$

a) $2_{45°} = 2(\cos 45° + i \,\text{sen}\, 45°) = 2\left(\frac{\sqrt{2}}{2} + i\,\frac{\sqrt{2}}{2}\right) = \sqrt{2} + \sqrt{2}\,i$

b) $3_{(\pi/6)} = 3\left(\cos\frac{\pi}{6} + i\,\text{sen}\,\frac{\pi}{6}\right) = 3\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$

c) $\sqrt{2}_{\,180°} = \sqrt{2}(\cos 180° + i\,\text{sen}\,180°) = \sqrt{2}(-1 + i \cdot 0) = -\sqrt{2}$

d) $17_{0°} = 17$

e) $1_{(\pi/2)} = \cos\frac{\pi}{2} + i\,\text{sen}\,\frac{\pi}{2} = i$

f) $5_{270°} = -5i$

g) $1_{150°} = \cos 150° + i\,\text{sen}\,150° = -\frac{\sqrt{3}}{2} + i\,\frac{1}{2} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

h) $4_{100°} = 4(\cos 100° + i\,\text{sen}\,100°) = 4(-0{,}17 + i \cdot 0{,}98) = -0{,}69 + 3{,}94i$

**16** **Dados os números complexos:**

$$z_1 = 2_{270°};\ z_2 = 4_{120°};\ z_3 = 3_{315°}$$

**calcula:**

**a)** $z_1 \cdot z_2$ **b)** $z_2 \cdot z_3$ **c)** $z_1 \cdot z_3$

**d)** $\dfrac{z_3}{z_1}$ **e)** $\dfrac{z_2}{z_1}$ **f)** $\dfrac{z_1 \cdot z_3}{z_2}$

**g)** $z_1^2$ **h)** $z_2^3$ **i)** $z_3^4$

a) $z_1 \cdot z_2 = 8_{30°}$ b) $z_2 \cdot z_3 = 12_{75°}$ c) $z_1 \cdot z_3 = 6_{225°}$

d) $\dfrac{z_3}{z_1} = 1{,}5_{45°}$ e) $\dfrac{z_2}{z_1} = 2_{-150°} = 2_{210°}$ f) $\dfrac{z_1 \cdot z_3}{z_2} = 1{,}5_{105°}$

g) $z_1^2 = 4_{180°}$ h) $z_2^3 = 64_{0°}$ i) $z_3^4 = 81_{180°}$

**17** **Expresa en forma polar e calcula:**

**a)** $(-1 - i)^5$ **b)** $\sqrt[4]{1 - \sqrt{3}i}$ **c)** $\sqrt[6]{64}$

**d)** $\sqrt[3]{8i}$ **e)** $\left(-2\sqrt{3} + 2i\right)^6$ **f)** $(3 - 4i)^3$

a) $(-1 - i)^5 = \left(\sqrt{2}_{\,225°}\right)^5 = 4\sqrt{2}_{\,1\,125°} = 4\sqrt{2}_{\,45°} = 4\sqrt{2}\left(\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right) = 4 + 4i$

b) $\sqrt[4]{1 - \sqrt{3}i} = \sqrt[4]{2_{300°}} = \sqrt[4]{2}_{(300° + 360°\, n)/4} = \sqrt[4]{2}_{75° + 90°\, n};\quad n = 0,\ 1,\ 2,\ 3$

Las cuatro raíces son:

$\sqrt[4]{2}_{75°}$ $\sqrt[4]{2}_{165°}$ $\sqrt[4]{2}_{255°}$ $\sqrt[4]{2}_{345°}$

c) $\sqrt[4]{64} = \sqrt[4]{64_{0°}} = \sqrt[4]{2^6}_{(360°\, k)/4} = 2\sqrt{2}_{\,90°\, k};\quad k = 0,\ 1,\ 2,\ 3$

Las cuatro raíces son:

$2\sqrt{2}_{\,0°} = 2\sqrt{2}$ $2\sqrt{2}_{\,90°} = 2\sqrt{2}\,i$ $2\sqrt{2}_{\,180°} = -2\sqrt{2}$ $2\sqrt{2}_{\,270°} = -2\sqrt{2}\,i$

d) $\sqrt[3]{8i} = \sqrt[3]{8_{90°}} = 2_{(90° + 360°\, k)/3} = 2_{30° + 120°\, k};\quad k = 0,\ 1,\ 2$

Las tres raíces son:

$2_{30°} = \sqrt{3} + i$ $2_{150°} = -\sqrt{3} + i$ $2_{270°} = -2i$

e) $\left(-2\sqrt{3} + 2i\right)^6 = (4_{150°})^6 = 4\,096_{900°} = 4\,096_{180°} = -4\,096$

f) $(3 - 4i)^3 = (5_{306°\ 52'})^3 = 125_{920°\ 36'} = 125_{200°\ 36'}$

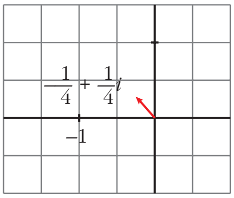
**18** **Calcula e representa graficamente o resultado:**

**a)** $\left(\dfrac{1-i}{\sqrt{3}+i}\right)^3$ **b)** $\sqrt[3]{\dfrac{1+i}{2-i}}$

a) $\left(\dfrac{1-i}{\sqrt{3}+i}\right)^3 = \left(\dfrac{\sqrt{2}_{315°}}{2_{30°}}\right)^3 = \left(\left(\dfrac{\sqrt{2}}{2}\right)_{285°}\right)^3 = \left(\dfrac{\sqrt{2}}{4}\right)_{855°} = \left(\dfrac{\sqrt{2}}{4}\right)_{135°} =$

$= \dfrac{\sqrt{2}}{4}(cos\ 135° + i\ sen\ 135°) =$

$= \dfrac{\sqrt{2}}{4}\left(-\dfrac{\sqrt{2}}{2} + i\dfrac{\sqrt{2}}{2}\right) = \dfrac{-1}{4} + \dfrac{1}{4}i$




b) $\sqrt[3]{\dfrac{1+i}{2-i}} = \sqrt[3]{\dfrac{(1+i)(2+i)}{(2-i)(2+i)}} = \sqrt[3]{\dfrac{1+3i}{5}} = \sqrt[3]{\dfrac{1}{5} + \dfrac{3}{5}i} =$

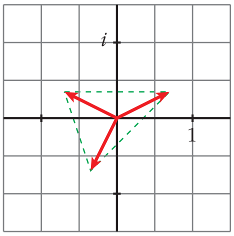
$= \sqrt[3]{\left(\dfrac{\sqrt{10}}{5}\right)_{71°\,34'}} = \left(\dfrac{\sqrt[6]{10}}{\sqrt[3]{5}}\right)_{(71°\,34' + 360°\,k)/3} = \sqrt[6]{\dfrac{2}{5}}_{23°\,51' + 120°\,k}$; $k = 0, 1, 2$

Las tres raíces son:

$\sqrt[6]{\dfrac{2}{5}}_{23°\,51'} = 0{,}785 + 0{,}347i$

$\sqrt[6]{\dfrac{2}{5}}_{143°\,51'} = -0{,}693 + 0{,}56i$

$\sqrt[6]{\dfrac{2}{5}}_{263°\,51'} = -0{,}092 - 0{,}853i$




**19** **Calcula e representa as solucións:**

**a)** $\sqrt[3]{4-4\sqrt{3}i}$ **b)** $\sqrt[4]{-16}$ **c)** $\sqrt[3]{8i}$

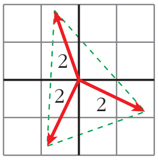
a) $\sqrt[3]{4-4\sqrt{3}\,i} = \sqrt[3]{8_{300°}} = 2_{(300° + 360°\,k)/3} = 2_{100° + 120°\,k}$; $k = 0, 1, 2$

Las tres raíces son:

$2_{100°} = -0{,}35 + 1{,}97i$

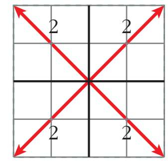
$2_{220°} = -1{,}53 - 1{,}26i$

$2_{340°} = 1{,}88 - 0{,}68i$



b) $\sqrt[4]{-16} = \sqrt[4]{16_{180°}} = 2_{(180° + 360°\,k)/4} = 2_{45° + 90°\,k};\quad k = 0, 1, 2, 3$

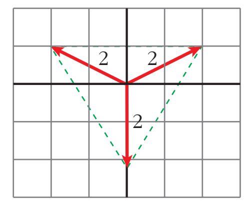
Las cuatro raíces son:



$2_{45°} = \sqrt{2} + \sqrt{2}\,i \qquad 2_{135°} = -\sqrt{2} + \sqrt{2}\,i$

$2_{225°} = -\sqrt{2} - \sqrt{2}\,i \qquad 2_{315°} = \sqrt{2} - \sqrt{2}\,i$

c) $\sqrt[3]{8i} = \sqrt[3]{8_{90°}} = 2_{(90° + 360°\,k)/3} = 2_{30° + 120°\,k};\quad k = 0, 1, 2$



Las tres raíces son:

$2_{30°} = \sqrt{3} + i \qquad 2_{150°} = -\sqrt{3} + i \qquad 2_{270°} = -2i$

## Páxina 163

**20** **Calcula pasando a forma polar:**

**a)** $(1 + i\sqrt{3})^5$ **b)** $(-1 - i\sqrt{3})^6(\sqrt{3} - i)$ **c)** $\sqrt[4]{-2 + 2\sqrt{3}\,i}$

**d)** $\dfrac{8}{(1 - i)^5}$ **e)** $\sqrt[6]{-64}$ **f)** $\sqrt{-1 - i}$

**g)** $\sqrt[3]{-i}$ **h)** $\sqrt{\dfrac{2 - 2i}{-3 + 3i}}$

a) $(1 + i\sqrt{3})^5 = (2_{60°})^5 = 32_{300°} = 32(\cos 300° + i \operatorname{sen} 300°) =$

$= 32\left(\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\,i\right) = 16 - 16\sqrt{3}\,i$

b) $(-1 - i\sqrt{3})^6(\sqrt{3} - i) = (2_{240°})^6\,(2_{330°}) = (64_{1\,440°})\,(2_{330°}) =$

$= (64_{0°})\,(2_{330°}) = 128_{330°} = 128(\cos 330° + i \operatorname{sen} 330°) =$

$= 128\left(\dfrac{\sqrt{3}}{2} + i\,\dfrac{-1}{2}\right) = 64\sqrt{3} - 64i$

c) $\sqrt[4]{-2 + 2\sqrt{3}\,i} = \sqrt[4]{4_{120°}} = \sqrt[4]{4}_{(120° + 360°\,k)/4} = \sqrt[4]{2^2}_{30° + 90°\,k} =$

$= \sqrt{2}_{30° + 90°\,k};\quad k = 0, 1, 2, 3$

Las cuatro raíces son:

$\sqrt{2}_{30°} = \dfrac{\sqrt{6}}{2} + \dfrac{\sqrt{2}}{2}\,i \qquad \sqrt{2}_{120°} = -\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{6}}{2}\,i$

$\sqrt{2}_{210°} = -\dfrac{\sqrt{6}}{2} - \dfrac{\sqrt{2}}{2}\,i \qquad \sqrt{2}_{300°} = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{6}}{2}\,i$

d) $\frac{8}{(1-i)^5} = \frac{8_{0°}}{(\sqrt{2}_{315°})^5} = \frac{8_{0°}}{4\sqrt{2}_{1575°}} = \frac{8_{0°}}{4\sqrt{2}_{135°}} = \left(\frac{8}{4\sqrt{2}}\right)_{-135°} = \left(\frac{2}{\sqrt{2}}\right)_{225°} =$

$= \sqrt{2}_{225°} = \sqrt{2}(cos\ 225° + i\ sen\ 225°) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = -1 - i$

e) $\sqrt[6]{-64} = \sqrt[6]{64_{180°}} = \sqrt[6]{2^6}_{(180° + 360° k)/6} = 2_{30° + 60° k}$; $k = 0, 1, 2, 3, 4, 5$

Las seis raíces son:

$2_{30°} = \sqrt{3} + i$ $\quad 2_{90°} = 2i$ $\quad 2_{150°} = -\sqrt{3} + i$

$2_{210°} = -\sqrt{3} - i$ $\quad 2_{270°} = -2$ $\quad 2_{330°} = \sqrt{3} - i$

f) $\sqrt{-1-i} = \sqrt{\sqrt{2}_{225°}} = \sqrt[4]{2}_{(225° + 360° k)/2} = \sqrt[4]{2}_{112°\ 30' + 180° k}$; $k = 0, 1$

Las dos raíces son:

$\sqrt[4]{2}_{112°\ 30'} = -0,46 + 1,1i$ $\quad \sqrt[4]{2}_{292°\ 30'} = 0,46 - 1,1i$

g) $\sqrt[3]{-i} = \sqrt[3]{1_{270°}} = 1_{(270° + 360° k)/3} = 1_{90° + 120° k}$; $k = 0, 1, 2$

Las tres raíces son:

$1_{90°} = i$ $\quad 1_{210°} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$ $\quad 1_{330°} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

h) $\sqrt{\frac{2-2i}{-3+3i}} = \sqrt{\frac{2\sqrt{2}_{315°}}{3\sqrt{2}_{135°}}} = \left(\frac{2}{3}\right)_{180°} = \left(\sqrt{\frac{2}{3}}\right)_{(180° + 360° k)/2} =$

$= \left(\sqrt{\frac{2}{3}}\right)_{90° + 180° k}$; $k = 0, 1$

Las dos raíces son:

$\left(\sqrt{\frac{2}{3}}\right)_{90°} = \sqrt{\frac{2}{3}}i$ $\quad \left(\sqrt{\frac{2}{3}}\right)_{270°} = -\sqrt{\frac{2}{3}}i$

21 **Expresa en forma polar $z$, o oposto $-z$, e o conxugado $\overline{z}$ en cada un destes casos:**

**a) $z = 1 - \sqrt{3}i$** **b) $z = -2 - 2i$** **c) $z = -2\sqrt{3} + 2i$**

a) $z = 1 - \sqrt{3}i = 2_{300°}$; $-z = -1 + \sqrt{3}i = 2_{120°}$; $\overline{z} = 1 + \sqrt{3}i = 2_{60°}$

b) $z = -2 - 2i = 2\sqrt{2}_{225°}$; $-z = 2 + 2i = 2\sqrt{2}_{45°}$; $\overline{z} = -2 + 2i = 2\sqrt{2}_{135°}$

c) $z = -2\sqrt{3} + 2i = 4_{150°}$; $-z = 2\sqrt{3} - 2i = 4_{330°}$; $\overline{z} = -2\sqrt{3} - 2i = 4_{210°}$

**22** **Representa os polígonos regulares que teñen por vértices os afixos das seguintes raíces:**

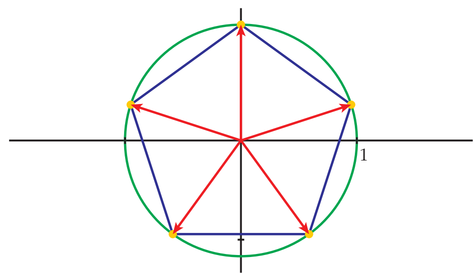
**a)** $\sqrt[5]{i}$ **b)** $\sqrt[6]{-1}$ **c)** $\sqrt[4]{2\sqrt{3} + 2i}$

a) $\sqrt[5]{i} = \sqrt[5]{1_{90°}} = 1_{(90° + 360°\,k)/5} = 1_{18° + 72°\,k}$; $k = 0, 1, 2, 3, 4$

Las cinco raíces son:

$1_{18°}$ $1_{90°}$ $1_{162°}$ $1_{234°}$ $1_{306°}$

Representación del polígono (pentágono):

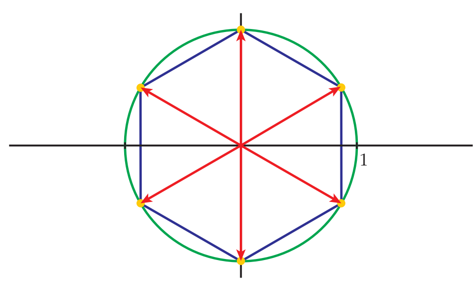



b) $\sqrt[6]{-1} = \sqrt[6]{1_{180°}} = 1_{(180° + 360°\,k)/6} = 1_{30° + 60°\,k}$; $k = 0, 1, 2, 3, 4, 5$

Las seis raíces son:

$1_{30°}$ $1_{90°}$ $1_{150°}$ $1_{210°}$ $1_{270°}$ $1_{330°}$

Representación del polígono (hexágono):

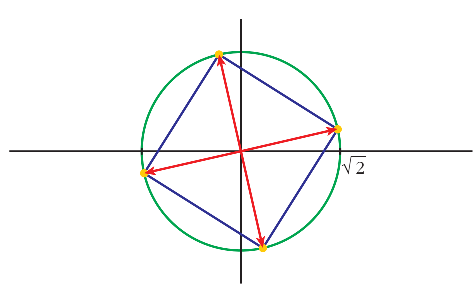



c) $\sqrt[4]{2\sqrt{3} + 2i} = \sqrt[4]{4_{30°}} = \sqrt[4]{2^2}_{(30° + 360°\,k)/4} = \sqrt{2}_{7°\,30' + 90°\,k}$; $k = 0, 1, 2, 3$

Las cuatro raíces son:

$\sqrt{2}_{7°\,30'}$ $\sqrt{2}_{97°\,30'}$ $\sqrt{2}_{187°\,30'}$ $\sqrt{2}_{277°\,30'}$

Representación del polígono (cuadrado):



## Ecuacións e sistemas en $\mathbb{C}$

**23** **Resolve as seguintes ecuacións e expresa as solucións en forma binómica:**

**a)** $z^2 + 4 = 0$ **b)** $z^2 + z + 4 = 0$

**c)** $z^2 + 3z + 7 = 0$ **d)** $z^2 - z + 1 = 0$

a) $z^2 + 4 = 0 \rightarrow z^2 = -4 \rightarrow z = \pm\sqrt{-4} = \pm 2i$

$z_1 = -2i, \; z_2 = 2i$

b) $z^2 + z + 4 = 0 \rightarrow z = \dfrac{-1 \pm \sqrt{1-16}}{2} = \dfrac{-1 \pm \sqrt{-15}}{2} = \dfrac{-1 \pm \sqrt{15}\,i}{2}$

$z_1 = -\dfrac{1}{2} - \dfrac{\sqrt{15}}{2}i, \; z_2 = -\dfrac{1}{2} + \dfrac{\sqrt{15}}{2}i$

c) $z^2 + 3z + 7 = 0 \rightarrow z = \dfrac{-3 \pm \sqrt{9-28}}{2} = \dfrac{-3 \pm \sqrt{-19}}{2} = \dfrac{-3 \pm \sqrt{19}\,i}{2}$

$z_1 = -\dfrac{3}{2} - \dfrac{\sqrt{19}}{2}i, \; z_2 = -\dfrac{3}{2} + \dfrac{\sqrt{19}}{2}i$

d) $z^2 - z + 1 = 0 \rightarrow z = \dfrac{1 \pm \sqrt{1-4}}{2} = \dfrac{1 \pm \sqrt{-3}}{2} = \dfrac{1 \pm \sqrt{3}\,i}{2}$

$z_1 = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i, \; z_2 = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

**24** **Resolve as ecuacións:**

**a)** $z^5 + 32 = 0$ **b)** $iz^3 - 27 = 0$

**c)** $z^3 + 8i = 0$ **d)** $iz^4 + 4 = 0$

a) $z^5 + 32 = 0 \rightarrow z^5 = -32$

$z = \sqrt[5]{-32} = \sqrt[5]{32_{180°}} = 2_{(180° + 360°\,k)/5} = 2_{36° + 72°\,k}; \; k = 0, 1, 2, 3, 4$

Las cinco raíces son:

$2_{36°} \quad 2_{108°} \quad 2_{180°} \quad 2_{252°} \quad 2_{324°}$

b) $iz^3 - 27 = 0 \rightarrow z^3 + 27i = 0 \rightarrow z^3 = -27i$

$z = \sqrt[3]{-27i} = \sqrt[3]{27_{270°}} = 3_{(270° + 360°\,k)/3} = 3_{90° + 120°\,k}; \; k = 0, 1, 2$

Las tres raíces son:

$3_{90°} \quad 3_{210°} \quad 3_{330°}$

c) $z^3 + 8i = 0 \rightarrow z = \sqrt[3]{-8i} = \sqrt[3]{8_{270°}} = 2_{(270° + 360°\,k)/3} = 2_{90° + 120°\,k}; \; k = 0, 1, 2$

Las tres raíces son:

$2_{90°} = 2i \quad 2_{210°} = -\sqrt{3} - i \quad 2_{330°} = \sqrt{3} - i$

d) $iz^4 + 4 = 0 \rightarrow z^4 - 4i = 0 \rightarrow z^4 = 4i$

$z = \sqrt[4]{4i} = \sqrt[4]{4_{90°}} = \sqrt{2}_{(90° + 360°\, k)/4} = \sqrt{2}_{22°\, 30' + 90°\, k}$; $k = 0, 1, 2, 3$

Las cuatro raíces son:

$\sqrt{2}_{22°\, 30'} = 1{,}3 + 0{,}5i$ $\quad\quad$ $\sqrt{2}_{112°\, 30'} = -0{,}5 + 1{,}3i$

$\sqrt{2}_{202°\, 30'} = -1{,}3 - 0{,}5i$ $\quad\quad$ $\sqrt{2}_{292°\, 30'} = 0{,}5 - 1{,}3i$

**25** **Resolve as seguintes ecuacións en $\mathbb{C}$ :**

**a)** $z^2 + 4i = 0$ $\quad\quad$ **b)** $z^2 - 2z + 5 = 0$

**c)** $2z^2 + 10 = 0$ $\quad\quad$ **d)** $z^4 + 13z^2 + 36 = 0$

a) $z^2 + 4i = 0 \rightarrow z^2 = -4i \rightarrow z = \sqrt{-4i} = \sqrt{4_{270°}} \rightarrow z = 2_{(270° + 360°\, k)/2}$; $k = 0,1$

$z_1 = 2_{135°}$, $z_2 = 2_{315°}$

b) $z^2 - 2z + 5 = 0 \rightarrow z = \dfrac{2 \pm \sqrt{4 - 20}}{2} = \dfrac{2 \pm \sqrt{-16}}{2} = \dfrac{2 \pm 4i}{2} = 1 \pm 2i$

$z_1 = 1 - 2i$, $z_2 = 1 + 2i$

c) $2z^2 + 10 = 0 \rightarrow 2z^2 = -10 \rightarrow z^2 = -5 \rightarrow z = \pm\sqrt{5}\, i$

$z_1 = -\sqrt{5}\, i$, $z_2 = \sqrt{5}\, i$

d) $z^4 + 13z^2 + 36 = 0$

$z^2 = t$

$t^2 + 13t + 36 = 0$

$t = \dfrac{-13 \pm \sqrt{169 - 144}}{2} = \dfrac{-13 \pm 5}{2} \begin{cases} t = -4 \\ t = -9 \end{cases}$

$z^2 = -4 \rightarrow z = \pm 2i$

$z^2 = -9 \rightarrow z = \pm 3i$

Las soluciones son: $2i = 2_{90°}$; $-2i = 2_{270°}$; $3i = 3_{90°}$; $-3i = 3_{270°}$

**26** **Obtén as catro solucións das seguintes ecuacións:**

**a)** $z^4 - 1 = 0$ $\quad\quad$ **b)** $z^4 + 16 = 0$ $\quad\quad$ **c)** $z^4 - 8z = 0$

a) $z^4 - 1 = 0 \rightarrow z^4 = 1 \rightarrow z = \sqrt[4]{1} = \sqrt[4]{1_{0°}} = 1_{360°\, k/4} = 1_{90°\, k}$; $k = 0, 1, 2, 3$

Las cuatro raíces son:

$1_{0°} = 1$ $\quad$ $1_{90°} = i$ $\quad$ $1_{180°} = -1$ $\quad$ $1_{270°} = -i$

b) $z^4 + 16 = 0 \rightarrow z^4 = -16 \rightarrow z = \sqrt[4]{-16} = \sqrt[4]{16_{180°}} = 2_{(180° + 360°\, k)/4} =$

$= 2_{45° + 90°\, k}$; $k = 0, 1, 2, 3$

Las cuatro raíces son:

$2_{45°} = \sqrt{2} + \sqrt{2}\,i$ $\quad$ $2_{135°} = -\sqrt{2} + \sqrt{2}\,i$

$2_{225°} = -\sqrt{2} - \sqrt{2}\,i$ $\quad$ $2_{315°} = \sqrt{2} - \sqrt{2}\,i$

c) $z^4 - 8z = 0 \rightarrow z\,(z^3 - 8) = 0 \begin{cases} z = 0 \\ z = \sqrt[3]{8} \end{cases}$

$\sqrt[3]{8} = \sqrt[3]{8_{0°}} = 2_{(360°\,k)/3} = 2_{120°\,k};\ \ k = 0,\ 1,\ 2$

Las soluciones de la ecuación son:

$0 \qquad 2_{0°} = 2 \qquad 2_{120°} = -1 + \sqrt{3}\,i \qquad 2_{240°} = -1 - \sqrt{3}\,i$

**27** **Calcula os números complexos $z$ e $w$ que verifican cada un destes sistemas de ecuacións:**

**a)** $\begin{cases} z + w = -1 + 2i \\ z - w = -3 + 4i \end{cases}$ $\qquad$ **b)** $\begin{cases} z + 2w = 2 + i \\ iz + w = 5 + 5i \end{cases}$

a) $\left.\begin{array}{l} z + w = -1 + 2i \\ z - w = -3 + 4i \end{array}\right\}$ Sumando miembro a miembro:

$2z = -4 + 6i \rightarrow z = -2 + 3i$

$w = (-1 + 2i) - (-2 + 3i) = 1 - i$

*Solución*: $z = -2 + 3i;\ \ w = 1 - i$

b) $\left.\begin{array}{l} z + 2w = 2 + i \\ iz + w = 5 + 5i \end{array}\right\}$ Multiplicamos por –2 la 2.ª ecuación y sumamos:

$\left.\begin{array}{l} z + 2w = 2 + i \\ -2iz - 2w = -10 - 10i \end{array}\right\}$ $(1 - 2i)z = -8 - 9i \rightarrow z = \dfrac{-8 - 9i}{1 - 2i} = 2 - 5i$

$w = \dfrac{2 + i - (2 - 5i)}{2} = \dfrac{6i}{2} = 3i$

*Solución*: $z = 2 - 5i;\ \ w = 3i$

## PARA RESOLVER

**28** **Calcula $m$ para que o número complexo $3 - mi$ teña o mesmo módulo ca $2\sqrt{5} + \sqrt{5}\,i$.**

$\left.\begin{array}{l} |3 - mi| = \sqrt{9 + m^2} \\ |2\sqrt{5} + \sqrt{5}\,i| = 5 \end{array}\right\}$ $\begin{array}{l} \sqrt{9 + m^2} = 5 \rightarrow 9 + m^2 = 25 \rightarrow m^2 = 16 \\ m = \pm 4 \end{array}$

Hay dos posibilidades: $m = -4$ y $m = 4$

**29** **Calcula dous números complexos tales que o seu cociente sexa 3, a suma dos seus argumentos π/3, e a suma dos seus módulos 8.**

☛ ***Chámalles $r_\alpha$ e $s_\beta$ e escribe as ecuacións que os relacionan:***

***$\frac{r_\alpha}{s_\beta} = 3_{0°}$ (0° é o argumento do cociente, $\alpha - \beta = 0°$); $r + s = 8$ e $\alpha + \beta = \frac{\pi}{3}$.***

$$\frac{r}{s} = 3$$

$$r + s = 8$$

$$\alpha + \beta = \frac{\pi}{3}$$

$$\alpha - \beta = 0°$$

Hallamos sus módulos:

$$\left.\begin{array}{l} \frac{r}{s} = 3 \\ r + s = 8 \end{array}\right\} \begin{array}{l} r = 3s \\ 3s + s = 8;\ \ 4s = 8;\ \ s = 2;\ \ r = 6 \end{array}$$

Hallamos sus argumentos:

$$\left.\begin{array}{l} \alpha + \beta = \frac{\pi}{3} \\ \alpha - \beta = 0 \end{array}\right\} \alpha = \beta;\ \ 2\beta = \frac{\pi}{3};\ \ \beta = \frac{\pi}{6};\ \ \alpha = \frac{\pi}{6}$$

Los números serán: $6_{\pi/6}$ y $2_{\pi/6}$

**30** **O produto de dous números complexos é $2_{90°}$ e o cubo do primeiro dividido polo outro é $(1/2)_{0°}$. Determínaos.**

Llamamos a los números: $z = r_\alpha$ y $w = s_\beta$

$$r_\alpha \cdot s_\beta = 2_{90°} \begin{cases} r \cdot s = 2 \\ \alpha + \beta = 90° \end{cases}$$

$$\frac{(r_\alpha)^3}{s_\beta} = \left(\frac{1}{2}\right)_{0°} \begin{cases} r^3/s = \frac{1}{2} \\ 3\alpha - \beta = 90° \end{cases}$$

$$\left.\begin{array}{l} r \cdot s = 2 \\ \frac{r^3}{s} = \frac{1}{2} \end{array}\right\} \left.\begin{array}{l} r \cdot s = 2 \\ s = 2r^3 \end{array}\right\} r \cdot 2r^3 = 2 \rightarrow r^4 = 1 \rightarrow r = \begin{cases} 1 \rightarrow s = 2 \cdot 1^3 = 2 \\ -1 \text{ (no vale)} \end{cases}$$

$$\left.\begin{array}{l} \alpha + \beta = 90° \\ 3\alpha - \beta = 0° \end{array}\right\} \rightarrow 4\alpha = 90° + 360°\,k \rightarrow$$

$$\rightarrow \alpha = \frac{90° + 360°k}{4},\quad k = 0, 1, 2, 3$$

$$\beta = 90° - \alpha$$

Hay cuatro soluciones:

$z_1 = 1_{22° 30'} \rightarrow w_1 = 2z_1^3 = 2 \cdot 1_{67° 30'} = 2_{67° 30'}$

$z_2 = 1_{112° 30'} \rightarrow w_2 = 2_{337° 30'}$

$z_3 = 1_{202° 30'} \rightarrow w_3 = 2_{607° 30'} = 2_{247° 30'}$

$z_4 = 1_{292° 30'} \rightarrow w_4 = 2_{877° 30'} = 2_{157° 30'}$

**31** **O produto de dous números complexos é $-8$ e o primeiro é igual ao cadrado do segundo. Calcúlaos.**

$$\left.\begin{array}{l} z \cdot w = -8 \\ z = w^2 \end{array}\right\} w^3 = -8$$

$w = \sqrt[3]{-8} = \sqrt[3]{8_{180°}} = 2_{(180° + 360° k)/3} = 2_{60° + 120° k};\ k = 0, 1, 2$

Hay tres soluciones:

$w_1 = 2_{60°} \rightarrow z_1 = 4_{120°}$

$w_2 = 2_{180°} \rightarrow z_2 = 4_{0°} = 4$

$w_3 = 2_{300°} \rightarrow z_3 = 4_{600°} = 4_{240°}$

**32** **De dous números complexos sabemos que:**

- **Teñen o mesmo módulo, igual a 2.**
- **Os seus argumentos suman $17\pi/6$.**
- **O primeiro é oposto do segundo.**

**Cales son eses números?**

Llamamos a los números: $z = r_\alpha$ y $w = s_\beta$

Tenemos que:

$$\left.\begin{array}{l} r = s = 2 \\ \alpha + \beta = \dfrac{17\pi}{6} \end{array}\right\} \rightarrow 2\alpha = \frac{17\pi}{6} + \pi \rightarrow \alpha = \frac{23}{12}\pi \rightarrow \beta = \frac{23}{12}\pi - \pi = \frac{11}{12}\pi$$

Por tanto, los números son: $1_{23\pi/12}$ y $2_{11\pi/12}$; o bien $1_{11\pi/12}$ y $2_{23\pi/12}$

**33** **Calcula *cos* 75° e *sen* 75° mediante o produto $1_{30°} \cdot 1_{45°}$.**

$1_{30°} \cdot 1_{45°} = 1_{75°} = \cos 75° + i \operatorname{sen} 75°$

$1_{30°} \cdot 1_{45°} = (\cos 30° + i \operatorname{sen} 30°)(\cos 45° + i \operatorname{sen} 45°) = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) =$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{6}}{4}i + \frac{\sqrt{2}}{4}i - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4} + \frac{\sqrt{6} + \sqrt{2}}{4}i$$

Por tanto:

$$\cos 75° = \frac{\sqrt{6} - \sqrt{2}}{4} \qquad \operatorname{sen} 75° = \frac{\sqrt{6} + \sqrt{2}}{4}$$

**34** **Calcula as razóns trigonométricas de 15º se coñeces as de 45º e as de 30º mediante o cociente $1_{45°} : 1_{30°}$.**

$1_{45°} : 1_{30°} = 1_{15°} = \cos 15° + i \operatorname{sen} 15°$

$$\frac{1_{45°}}{1_{30°}} = \frac{\cos 45° + i \operatorname{sen} 45°}{\cos 30° + i \operatorname{sen} 30°} = \frac{\sqrt{2}/2 + i(\sqrt{2}/2)}{\sqrt{3}/2 + i(1/2)} = \frac{\sqrt{2} + i\sqrt{2}}{\sqrt{3} + i} =$$

$$= \frac{(\sqrt{2} + i\sqrt{2})(\sqrt{3} - i)}{(\sqrt{3} + i)(\sqrt{3} - i)} = \frac{\sqrt{6} - \sqrt{2}i + \sqrt{6}i + \sqrt{2}}{3 + 1} = \frac{\sqrt{6} + \sqrt{2}}{4} + \frac{\sqrt{6} + \sqrt{2}}{4}i$$

Por tanto:

$$\cos 15° = \frac{\sqrt{6} + \sqrt{2}}{4} \qquad \operatorname{sen} 15° = \frac{\sqrt{6} - \sqrt{2}}{4}$$

**35** **Para que valores de $x$ é imaxinario puro o cociente $\frac{x - 4i}{x + i}$?**

$$\frac{x - 4i}{x + i} = \frac{(x - 4i)(x - i)}{(x + i)(x - i)} = \frac{x^2 - 4}{x^2 + 1} + \frac{-5x}{x^2 + 1}i$$

Para que sea imaginario puro, ha de ser:

$$\frac{x^2 - 4}{x^2 + 1} = 0 \rightarrow x^2 - 4 = 0 \begin{cases} x = 2 \\ x = -2 \end{cases}$$

**36** **Calcula, en función de $x$, o módulo de $z = \frac{1 + xi}{1 - xi}$.**

**Demostra que $|z| = 1$ para calquera valor de $x$.**

$$|z| = \left|\frac{1 + xi}{1 - xi}\right| = \frac{\sqrt{1 + x^2}}{\sqrt{1 + x^2}} = 1$$

O bien:

$$z = \frac{1 + xi}{1 - xi} = \frac{(1 + xi) + (1 + xi)}{(1 - xi)(1 + xi)} = \frac{1 - x^2 + 2xi}{1 + x^2} = \frac{1 - x^2}{1 + x^2} + \frac{2x}{1 + x^2}i$$

$$|z| = \sqrt{\left(\frac{1 - x^2}{1 + x^2}\right)^2 + \left(\frac{2x}{1 + x^2}\right)^2} = \sqrt{\frac{1 + x^4 - 2x^2 + 4x^2}{(1 + x^2)^2}} = \sqrt{\frac{x^4 + 2x^2 + 1}{(1 + x^2)^2}} =$$

$$= \sqrt{\frac{(1 + x^2)^2}{(1 + x^2)^2}} = \sqrt{1} = 1$$

**37** **Calcula $x$ para que o número complexo que obtemos ao dividir $\frac{x + 2i}{4 - 3i}$ estea representado na bisectriz do primeiro cuadrante.**

☛ ***Para que $a + bi$ estea na bisectriz do primeiro cuadrante, debe ser $a = b$.***

$$\frac{x + 2i}{4 - 3i} = \frac{(x + 2i)(4 + 3i)}{(4 - 3i)(4 + 3i)} = \frac{4x + 3xi + 8i - 6}{16 + 9} = \frac{4x - 6}{25} + \frac{3x + 8}{25}i$$

Ha de ser:

$$\frac{4x-6}{25}=\frac{3x+8}{25} \rightarrow 4x-6=3x+8 \Rightarrow x=14$$

## Páxina 164

**38** **Calcula dous números complexos conxugados cuxa suma é 8 e a suma dos módulos é 10.**

$$\left.\begin{array}{l} z+\bar{z}=8 \\ |z|+|\bar{z}|=10 \end{array}\right\} \text{ Como } |z|=|\bar{z}| \Rightarrow |z|=5$$

Si llamamos:

$z=a+bi \rightarrow \bar{z}=a-bi$

$z+\bar{z}=a+bi+a-bi=2a=8 \rightarrow a=4$

$|z|=|\bar{z}|=\sqrt{a^2+b^2}=\sqrt{16+b^2}=5 \rightarrow 16+b^2=25 \rightarrow$

$\rightarrow b^2=9 \rightarrow b=\pm\sqrt{9}=\pm 3$

Hay dos soluciones:

$z_1=4+3i \rightarrow \bar{z}_1=4-3i$

$z_2=4-3i \rightarrow \bar{z}_2=4+3i$

**39** **A suma de dous números complexos é $3+i$. A parte real do primeiro é 2, e o produto dos dous é un número real. Determínaos.**

Llamamos $z=a+bi$ y $w=c+di$

Tenemos que:

$$\left\{\begin{array}{l} z+w=3+i \\ a=2 \rightarrow c=1 \end{array}\right. \qquad \left\{\begin{array}{l} a+c=3 \\ b+d=1 \end{array}\right.$$

$z \cdot w=(2+bi)(1+di)=2+2di+bi+bdi^2=(2-bd)+(2d+b)i$

Para que $z \cdot w$ sea un número real, ha de ser $2d+b=0$.

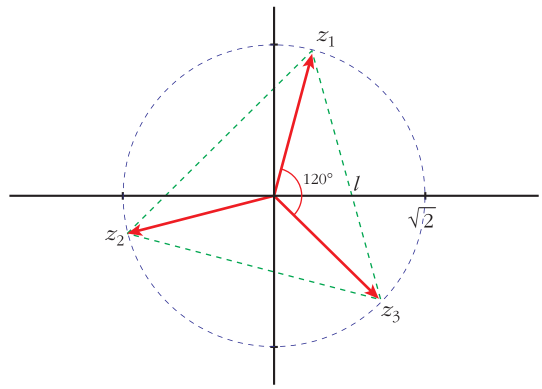
Por tanto, $\left.\begin{array}{l} b+d=1 \\ b+2d=0 \end{array}\right\} \begin{array}{l} d=-1 \\ b=2 \end{array}$ Los números son: $z=2+2i$; $w=1-i$

**40** **Representa graficamente os resultados que obteñas ao calcular $\sqrt[3]{-2-2i}$ e indica tamén o lado do triángulo que se forma ao unir eses tres puntos.**

$$\sqrt[3]{-2-2i}=\sqrt[3]{\sqrt{8}_{225°}}=\sqrt{2}_{(225°+360°k)/3}=\sqrt{2}_{75°+120°k}$$

Las tres raíces son:

$z_1=\sqrt{2}_{75°} \qquad z_2=\sqrt{2}_{195°} \qquad z_3=\sqrt{2}_{315°}$




Para hallar la longitud del lado, aplicamos el teorema del coseno:

$l^2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 - 2\sqrt{2} \cdot \sqrt{2} \cdot cos\ 120° = 2 + 2 - 4\left(-\frac{1}{2}\right) = 4 + 2 = 6$

$l = \sqrt{6}$

**41** **Os afixos das raíces cúbicas de $8i$ son os vértices dun triángulo equilátero. Compróbao.**

**Determinan o mesmo triángulo os afixos de $\sqrt[3]{-8i}$, $\sqrt[3]{8}$ ou $\sqrt[3]{-8}$? Representa graficamente eses catro triángulos que obtiveches.**

- $\sqrt[3]{8i} = \sqrt[3]{8_{90°}} = 2_{(90° + 360°\ k)/3} = 2_{30° + 120°\ k};\ \ k = 0, 1, 2$

  Las tres raíces son:

  $z_1 = 2_{30°} \qquad z_2 = 2_{150°} \qquad z_3 = 2_{270°}$

  Al tener el mismo módulo y formar entre ellos un ángulo de 120°, el triángulo que determinan es equilátero.

- $\sqrt[3]{-8i} = \sqrt[3]{8_{270°}} = 2_{(270° + 360°\ k)/3} = 2_{90° + 120°\ k};\ \ k = 0, 1, 2$

  Las tres raíces son:

  $z_1 = 2_{90°} \qquad z_2 = 2_{210°} \qquad z_3 = 2_{330°}$

- $\sqrt[3]{8} = \sqrt[3]{8_{0°}} = 2_{360°\ k/3} = 2_{120°\ k};\ \ k = 0, 1, 2$

  Las tres raíces son:

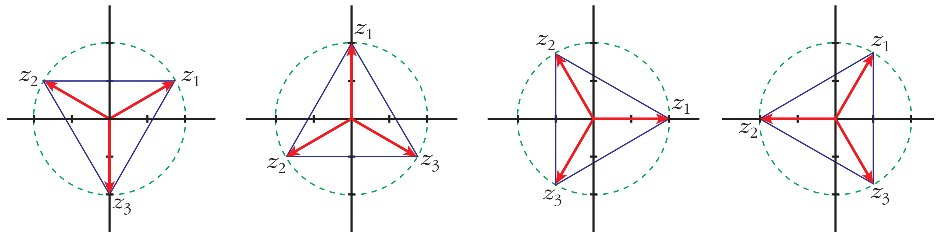
  $z_1 = 2_{0°} \qquad z_2 = 2_{120°} \qquad z_3 = 2_{240°}$

- $\sqrt[3]{-8} = \sqrt[3]{8_{180°}} = 2_{(180° + 360°\ k)/3} = 2_{60° + 120°\ k};\ \ k = 0, 1, 2$

  Las tres raíces son:

  $z_1 = 2_{60°} \qquad z_2 = 2_{180°} \qquad z_3 = 2_{300°}$

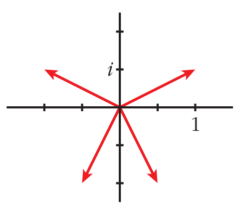
• Representación:



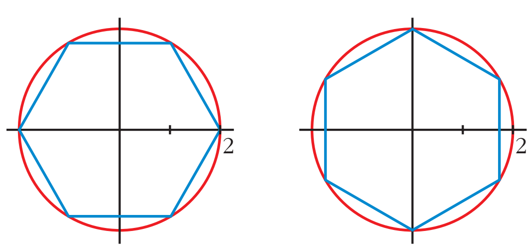

$\sqrt[3]{8i}$ $\sqrt[3]{-8i}$ $\sqrt[3]{8}$ $\sqrt[3]{-8}$

**42** **Poden ser $z_1 = 2 + i$, $z_2 = -2 + i$, $z_3 = -1 - 2i$ e $z_4 = 1 - 2i$, as raíces dun número complexo? Xustifica a resposta.**

No. Si fueran las cuatro raíces cuartas de un número complejo, formarían entre cada dos de ellas un ángulo de 90°; y ni siquiera forman el mismo ángulo, como vemos en la representación gráfica:




**43** **Calcula os números complexos que corresponden aos vértices destes hexágonos:**




• 1.$^{er}$ hexágono:

$z_1 = 2_{0°} = 2$ $z_2 = 2_{60°} = 1 + \sqrt{3}\,i$ $z_3 = 2_{120°} = -1 + \sqrt{3}\,i$

$z_4 = 2_{180°} = -2$ $z_5 = 2_{240°} = -1 - \sqrt{3}\,i$ $z_6 = 2_{300°} = 1 - \sqrt{3}\,i$

• 2.° hexágono:

$z_1 = 2_{30°} = \sqrt{3} + i$ $z_2 = 2_{90°} = 2i$ $z_3 = 2_{150°} = -\sqrt{3} + i$

$z_4 = 2_{210°} = -\sqrt{3} - i$ $z_5 = 2_{270°} = -2i$ $z_6 = 2_{330°} = \sqrt{3} - i$

**44** **Poden ser as raíces dun número complexo, $z$, os números $2_{28°}$, $2_{100°}$, $2_{172°}$, $2_{244°}$ e $2_{316°}$? En caso afirmativo, calcula $z$.**

☛ ***Comproba se o ángulo que forman cada dúas delas é o dun pentágono regular.***

$28° + 72° = 100°$ $\quad$ $100° + 72° = 172°$

$172° + 72° = 244°$ $\quad$ $244° + 72° = 316°$

Sí son las raíces quintas de un número complejo. Lo hallamos elevando a la quinta cualquiera de ellas:

$$z = (2_{28°})^5 = 32_{140°}$$

**45** **O número complexo $3_{40°}$ é vértice dun pentágono regular. Calcula os outros vértices e o número complexo cuxas raíces quintas son eses vértices.**

☛ ***Para obter os outros vértices podes multiplicar cada un por $1_{72°}$.***

Los otros vértices serán:

$$3_{112°} \qquad 3_{184°} \qquad 3_{256°} \qquad 3_{328°}$$

El número será:

$$z = (3_{40°})^5 = 243$$

**46** **Unha das raíces cúbicas dun número complexo $z$ é $1 + i$. Calcula $z$ e mais as outras raíces cúbicas.**

☛ ***Ten en conta que se $\sqrt[3]{z} = 1 + i \rightarrow z = (1 + i)^3$.***

$1 + i = \sqrt{2}_{45°}$

Las otras raíces cúbicas son:

$\sqrt{2}_{45° + 120°} = \sqrt{2}_{165°}$ $\qquad$ $\sqrt{2}_{165° + 120°} = \sqrt{2}_{285°}$

Hallamos $z$:

$$z = (1 + i)^3 = \left(\sqrt{2}_{45°}\right)^3 = \sqrt{8}_{135°} = \sqrt{8}\,(cos\ 135° + i\ sen\ 135°) =$$

$$= \sqrt{8}\left(-\frac{\sqrt{2}}{2} + i\,\frac{\sqrt{2}}{2}\right) = -2 + 2i$$

**47** **Escribe unha ecuación de segundo grao que teña por solucións $1 + i$ e $1 - i$.**

☛ ***Mira o exercicio resolto 1 da páxina 151.***

$$[x - (1 + i)]\,[x - (1 - i)] = x^2 - (1 - i)x - (1 + i)x + (1 + i)\,(1 - i) =$$

$$= x^2 - (1 - i + 1 + i)x + (1 - i^2) =$$

$$= x^2 - 2x + 2 = 0$$

**48** **Escribe unha ecuación de segundo grao cuxas solucións sexan:**

**a)** $5i$ **e** $-5i$

**b)** $2 - 3i$ **e** $2 + 3i$

a) $(x - 5i)(x + 5i) = 0$

$x^2 - 25i^2 = 0$

$x^2 + 25 = 0$

b) $[x - (2 - 3i)][x - (2 + 3i)] = [(x - 2) + 3i][(x - 2) - 3i] =$

$= (x - 2)^2 - (3i^2) = x^2 - 4x + 4 - 9i^2 =$

$= x^2 - 4x + 13 = 0$

**49** **Resolve os seguintes sistemas de ecuacións:**

**a)** $\begin{cases} z + w = -1 + 2i \\ iz + (1 - i)w = 1 + 3i \end{cases}$

**b)** $\begin{cases} z - w = 5 - 3i \\ (2 + i)z + iw = 3 - 3i \end{cases}$

a) Multiplicamos por $-i$ la primera ecuación:

$\left.\begin{array}{l} -iz - iw = i + 2 \\ iz + (1 - i)w = 1 + 3i \end{array}\right\}$ Sumamos miembro a miembro: $-iw + (1 - i)w = i + 2 + 1 + 3i \rightarrow (1 - 2i)w = 3 + 4i$

$$w = \frac{3 + 4i}{1 - 2i} = \frac{(3 + 4i)(1 + 2i)}{1^2 - 2i^2} = \frac{-5 + 10i}{5} = -1 + 2i$$

$z = -1 + 2i - w = -1 + 2i + 1 - 2i = 0$

*Solución*: $z = 0$; $w = -1 + 2i$

b) Multiplicamos por $i$ la primera ecuación:

$\left.\begin{array}{l} zi - wi = 5i + 3 \\ (2 + i)z + wi = 3 - 3i \end{array}\right\}$ Sumamos miembro a miembro: $zi + (2 + i)z = 5i + 3 + 3 - 3i \rightarrow (2 + 2i)z = 6 + 2i$

$$z = \frac{6 + 2i}{2 + 2i} = \frac{(6 + 2i)(2 - 2i)}{4 - 4i^2} = \frac{16 - 8i}{8} = 2 - i$$

$w = z - 5 + 3i = 2 - i - 5 + 3i = -3 + 2i$

*Solución*: $z = 2 - i$; $w = -3 + 2i$

## Interpretación gráfica de igualdades e desigualdades entre complexos

**50** **Representa.**

**a)** $Re\ z = 2$

**b)** $Im\ z = 1$

**c)** $Re\ z \le 0$

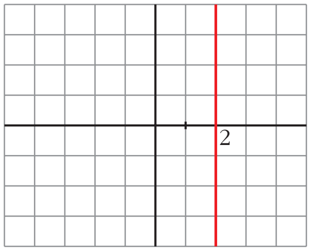
**d)** $-1 \le Im\ z \le 3$

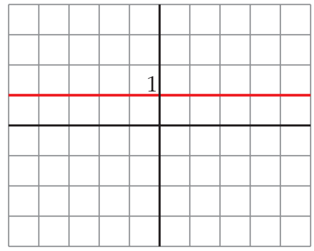
**e)** $-2 < Re\ z < 5$

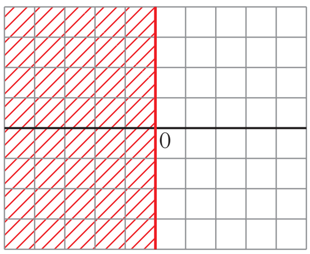
**f)** $|z| \le 3$

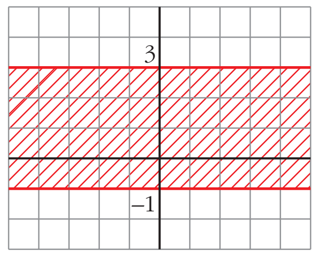
**g)** $Arg\ z = 45°$

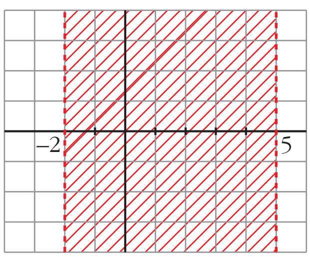
**h)** $0° \le Arg\ z \le 90°$

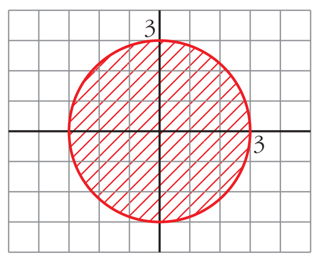
a)


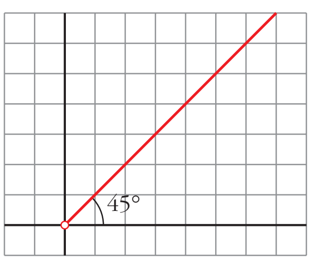

b)


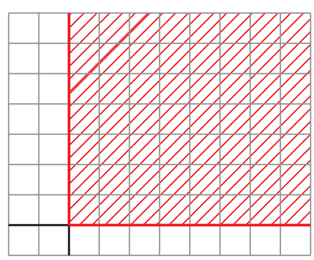

c)



d)



e)



f)



g)



h)


**51** **Representa os números complexos $z$ tales que $z + \overline{z} = -3$.**

☛ ***Escribe $z$ en forma binómica, súmalle o seu conxugado e representa a condición que obtés.***

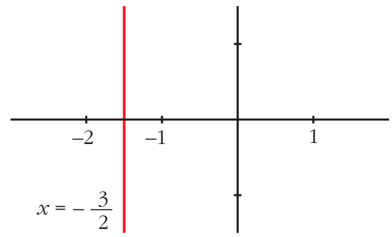
Llamamos $z = x + iy$

Entonces: $\overline{z} = x - iy$

Así:

$$z + \overline{z} = x + iy + x - iy = 2x = -3 \ \rightarrow \ x = -\frac{3}{2}$$

Representación:



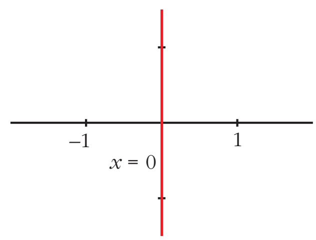

**52** **Representa os números complexos que verifican:**

**a)** $\bar{z} = -z$ **b)** $|z + \bar{z}| = 3$ **c)** $|z - \bar{z}| = 4$

a) $z = x + iy \rightarrow \bar{z} = x - iy$

$\bar{z} = -z \rightarrow x - iy = -x - iy \rightarrow 2x = 0 \rightarrow x = 0$ (es el eje imaginario)

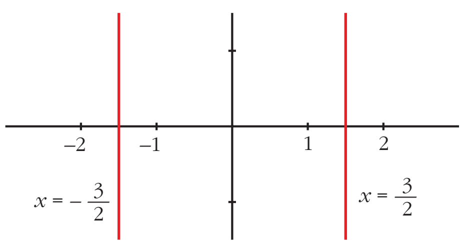
Representación:




b) $z + \bar{z} = x + iy + x - iy = 2x$

$|z + \bar{z}| = |2x| = 3 \left\langle \begin{array}{l} 2x = 3 \rightarrow x = 3/2 \\ 2x = -3 \rightarrow x = -3/2 \end{array} \right.$

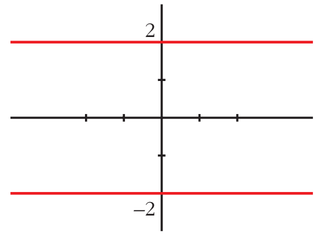
Representación:



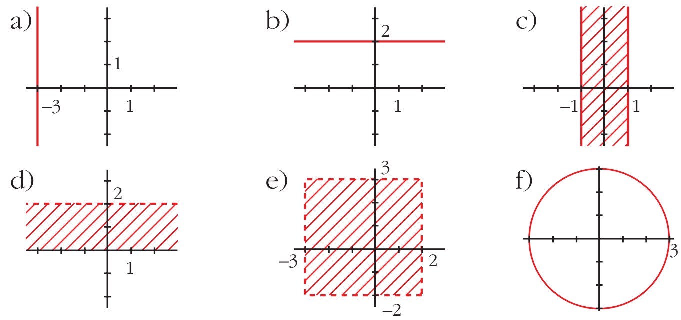

c) $z - \bar{z} = x + iy - z + iy = 2yi$

$|z - \bar{z}| = |2yi| = |2y| = 4 \left\langle \begin{array}{l} 2y = 4 \rightarrow y = 2 \\ 2y = -4 \rightarrow y = -2 \end{array} \right.$

Representación:



**53** **Escribe as condicións que deben cumprir os números complexos cuxa representación gráfica é a seguinte:**




☛ ***En a), b) e f) é unha igualdade. En c) e d), unha desigualdade. En e), dúas desigualdades.***

a) $Re\ z = -3$

b) $Im\ z = 2$

c) $-1 \leq Re\ z \leq 1$

d) $0 \leq Im\ z < 2$

e) $\begin{cases} -3 < Re\ z < 2 \\ -2 < Im\ z < 3 \end{cases}$

f) $|z| = 3$

## Páxina 165

### CUESTIÓNS TEÓRICAS

**54** **Pódese dicir que un número complexo é real se o seu argumento é 0?**

No, también son reales los números con argumento 180° (los negativos).

**55** **Se $z = r_\alpha$, que relación teñen con $z$ os números $r_{\alpha + 180°}$ e $r_{360° - \alpha}$?**

$r_{\alpha + 180°} = -z$ (opuesto de $z$)

$r_{360° - \alpha} = \overline{z}$ (conjugado de $z$)

**56** **Comproba que:**

**a)** $\overline{z + w} = \overline{z} + \overline{w}$ **b)** $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$ **c)** $\overline{kz} = k\,\overline{z}$, **con** $k \in \mathbb{R}$

$z = a + bi = r_\alpha \rightarrow \overline{z} = a - bi = r_{360° - \alpha}$

$w = c + di = r'_\beta \rightarrow \overline{w} = c - di = r'_{360° - \beta}$

a) $z + w = (a + c) + (b + d)i \rightarrow \overline{z + w} = (a + c) - (b + d)i$

$\overline{z} + \overline{w} = a - bi + c - di = (a + c) - (b + d)i = \overline{z + w}$

b) $z \cdot w = (r \cdot r')_{\alpha + \beta} \rightarrow \overline{z \cdot w} = (r \cdot r')_{360° - (\alpha + \beta)}$

$\overline{z} \cdot \overline{w} = (r \cdot r')_{360° - \alpha + 360° - \beta} = (r \cdot r')_{360° - (\alpha + \beta)} = \overline{z \cdot w}$

c) $kz = ka + kbi \rightarrow \overline{kz} = ka - kbi$

$k\overline{z} = ka - kbi = \overline{kz}$

**57** **Demostra que:**

$$\left| \frac{1}{z} \right| = \frac{1}{|z|}$$

$$\frac{1}{z} = \frac{1_{0°}}{r_\alpha} = \left(\frac{1}{r}\right)_{-\alpha} = \left(\frac{1}{r}\right)_{360° - \alpha} \rightarrow \left|\frac{1}{z}\right| = \frac{1}{r} = \frac{1}{|z|}$$

**58** **O produto de dous números complexos imaxinarios, pode ser real? Aclárao cun exemplo.**

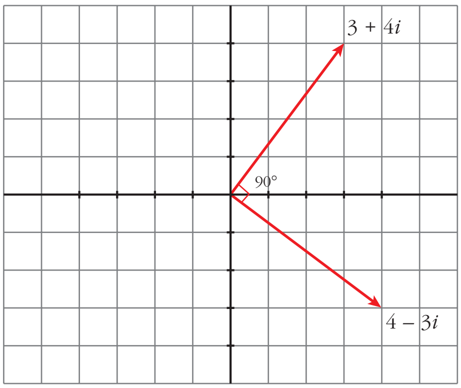
Sí. Por ejemplo:

$z = i, \ w = i$

$z \cdot w = i \cdot i = i^2 = -1 \in \mathbb{R}$

**59** **Representa o número complexo $z = 4 - 3i$. Multiplícao por $i$ e comproba que o resultado que obtés é o mesmo que se lle aplicas a $z$ un xiro de 90°.**

$iz = 4i - 3i^2 = 3 + 4i$




**60** **Que relación existe entre o argumento dun complexo e o do seu oposto?**

Se diferencian en 180°. Si el argumento del número es $\alpha$, el de su opuesto es:

$$180° + \alpha$$

**61** **Que condición debe cumprir un número complexo $z = a + bi$ para que $\bar{z} = \frac{1}{z}$?**

☛ ***Calcula $\frac{1}{z}$, e iguala a $a - bi$.***

$$\frac{1}{z} = \frac{1}{a + bi} = \frac{a - bi}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2} = a - bi$$

$$\left.\begin{array}{l} \frac{a}{a^2 + b^2} = a \\ \frac{-b}{a^2 + b^2} = -b \end{array}\right\} \begin{array}{l} \frac{a}{a} = a^2 + b^2 \rightarrow a^2 + b^2 = 1 \text{ (módulo 1)} \\ \text{Ha de tener módulo 1.} \end{array}$$

## PARA AFONDAR

**62** **Un pentágono regular con centro na orixe de coordenadas ten un dos seus vértices no punto $(\sqrt{2}, \sqrt{2})$. Calcula os outros vértices e a lonxitude do lado.**

El punto $(\sqrt{2}, \sqrt{2})$ corresponde al afijo del número complejo $z = \sqrt{2} + \sqrt{2}\,i = 2_{45°}$.

Para hallar los otros vértices, multiplicamos $z$ por $1_{72°}$:

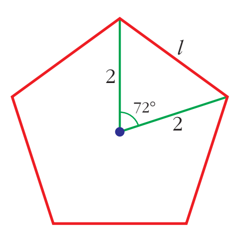
$z_2 = 2_{117°} = -0{,}91 + 1{,}78i$ $\quad$ $z_3 = 2_{189°} = -1{,}97 - 0{,}31i$

$z_4 = 2_{261°} = -0{,}31 - 1{,}97i$ $\quad$ $z_5 = 2_{333°} = 1{,}78 - 0{,}91i$

Los otros cuatro vértices serán:

(–0,91; 1,78) (–1,97; –0,31) (–0,31; –1,97) (1,78; –0,91)

Hallamos la longitud del lado aplicando el teorema del coseno:




$l^2 = 2^2 + 2^2 - 2 \cdot 2 \cdot cos\ 72°$

$l^2 = 4 + 4 - 4 \cdot 0{,}31$

$l^2 = 8 - 1{,}24$

$l^2 = 6{,}76$

$l = 2{,}6$ unidades

**63** **Se o produto de dous números complexos é –8 e dividindo o cubo dun deles entre o outro obtemos de resultado 2, canto valen o módulo e o argumento de cada un?**

$$\left.\begin{array}{l} z = r_\alpha \\ w = r'_\beta \\ -8 = 8_{180°} \\ 2 = 2_{0°} \end{array}\right\} \quad r_\alpha \cdot r'_\beta = (r \cdot r')_{\alpha + \beta} = 8_{180°} \;\rightarrow\; \begin{cases} r \cdot r' = 8 \\ \alpha + \beta = 180° \end{cases}$$

$$\frac{(r_\alpha)^3}{r'_\beta} = \frac{r^3{}_{3\alpha}}{r'_\beta} = \left(\frac{r^3}{r'}\right)_{3\alpha - \beta} = 2_{0°} \;\rightarrow\; \begin{cases} \dfrac{r^3}{r'} = 2 \\ 3\alpha - \beta = 0° \end{cases}$$

Así:

$$\left.\begin{array}{l} r \cdot r' = 8 \\ r^3 = 2r' \end{array}\right\} \quad \left.\begin{array}{l} r' = \dfrac{8}{r} \\ r' = \dfrac{r^3}{2} \end{array}\right\} \quad \frac{8}{r} = \frac{r^3}{2} \;\rightarrow\; 16 = r^4 \;\rightarrow \begin{cases} r = 2 \\ r' = 4 \end{cases}$$

$$\left.\begin{array}{l} \alpha + \beta = 180° \\ 3\alpha = \beta \end{array}\right\} \alpha + 3\alpha = 180° \;\rightarrow\; 4\alpha = 180° \;\rightarrow\; \begin{cases} \alpha = 45° \\ \beta = 135° \end{cases}$$

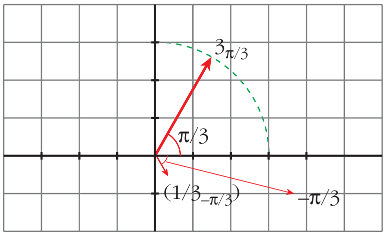
Por tanto: $z = 2_{45°}$, $w = 4_{135°}$

**64** **Calcula o inverso dos números complexos seguintes e representa graficamente o resultado que obteñas:**

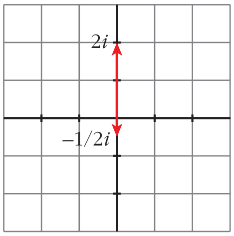
**a)** $3_{\pi/3}$ **b)** $2i$ **c)** $-1 + i$

**Que relación existe entre o módulo e o argumento dun número complexo e do inverso?**

a) $\dfrac{1}{3_{\pi/3}} = \dfrac{1_{0°}}{3_{\pi/3}} = \left(\dfrac{1}{3}\right)_{-\pi/3} = \left(\dfrac{1}{3}\right)_{5\pi/3}$

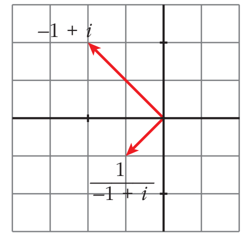



b) $\dfrac{1}{2i} = \dfrac{-i}{2} = \dfrac{-1}{2}\,i = \left(\dfrac{1}{2}\right)_{270°}$



c) $-1 + i = \sqrt{2}_{135°}$

$$\frac{1}{-1 + i} = \frac{1_{0°}}{\sqrt{2}_{135°}} = \left(\frac{1}{\sqrt{2}}\right)_{-135°} = \left(\frac{1}{\sqrt{2}}\right)_{225°} = -\frac{1}{2} - \frac{1}{2}\,i$$

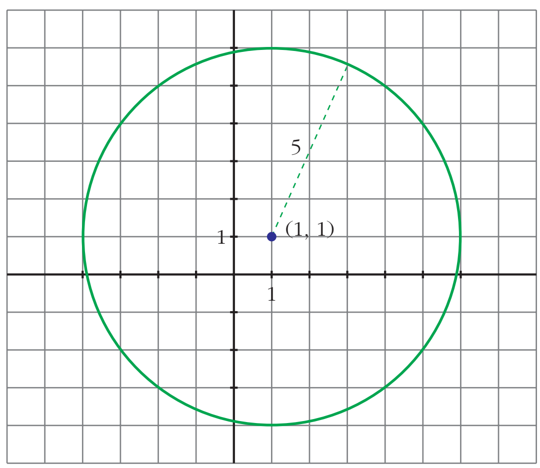



Si $z = r_\alpha$, entonces $\frac{1}{z} = \left(\frac{1}{r}\right)_{360° - \alpha}$

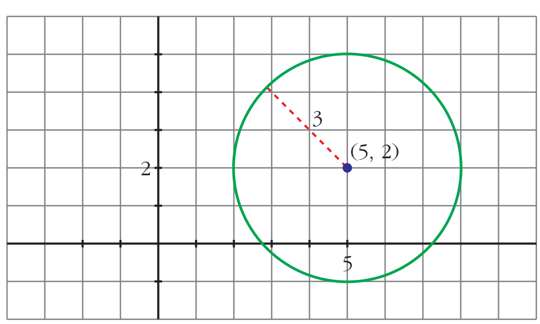
**65** **Representa graficamente as igualdades seguintes. Que figura se determina en cada caso?**

**a)** $|z - (1 + i)| = 5$ **b)** $|z - (5 + 2i)| = 3$

a) Circunferencia con centro en (1, 1) y radio 5.




b) Circunferencia de centro en (5, 2) y radio 3.




**66** **Escribe a condición que verifican todos os números complexos cuxos afixos estean na circunferencia de centro (1, 1) e raio 3.**

$|z - (1 + i)| = 3$

# AUTOAVALIACIÓN

**1.** **Efectúa.**

$$\frac{(3-2i)^2-(1+i)(2-i)}{-3+i}$$

$$\frac{(3-2i)^2-(1+i)(2-i)}{-3+i}=\frac{9+4i^2-12i-(2-i+2i-i^2)}{-3+i}=\frac{5-12i-3-i}{-3+i}=$$

$$=\frac{(2-13i)(-3-i)}{(-3+i)(-3-i)}=\frac{-6+13i^2-2i+39i}{9-i^2}=$$

$$=\frac{-19+37i}{10}=-\frac{19}{10}+\frac{37}{10}i$$

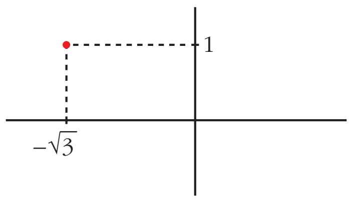
**2.** **Calcula $z$ e expresa os resultados en forma binómica.**

$$\sqrt[4]{z}=\frac{-\sqrt{3}+i}{\sqrt{2}\,i}$$

$$z=\left(\frac{-\sqrt{3}+i}{\sqrt{2}\,i}\right)^4$$

Pasamos numerador y denominador a forma polar:

$$-\sqrt{3}+i \begin{cases} r=\sqrt{(-\sqrt{3})^2+1^2}=2 \\ tg\ \alpha=-\dfrac{1}{\sqrt{3}} \ \rightarrow\ \alpha=150° \end{cases}$$




$$\sqrt{2}i \ \rightarrow\ \sqrt{2}_{90°}$$

$$z=\left(\frac{2_{150°}}{\sqrt{2}_{90°}}\right)^4=(\sqrt{2}_{60°})^4=4_{240°}\ \rightarrow\ z=4\,(cos\ 240°+i\ sen\ 240°)$$

$$z=4\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)=-2-2\sqrt{3}\,i$$

**3.** **Calcula $a$ e $b$ para que se verifique a igualdade:**

$$5(a-2i)=(3+i)(b-i)$$

$$5a-10i=3b-i^2-3i+bi\ \rightarrow\ 5a-10i=3b+1+(-3+b)i$$

Igualando las componentes $\begin{cases} 5a=3b+1 \\ -10=-3+b \end{cases}\ \rightarrow\ b=-7,\ \ a=-4$

**4.** **Resolve a ecuación:** $z^2 - 10z + 29 = 0$

$$z = \frac{10 \pm \sqrt{-16}}{2} = \frac{10 \pm 4i}{2} \begin{cases} z_1 = 5 + 2i \\ z_2 = 5 - 2i \end{cases}$$

*Solucións:* $z_1 = 5 + 2i, \; z_2 = 5 - 2i$

**5.** **Calcula o valor que debe tomar** $x$ **para que o módulo de** $\frac{x + 2i}{1 - i}$ **sexa igual a 2.**

$$\frac{x + 2i}{1 - i} = \frac{(x + 2i)(1 + i)}{(1 - i)(1 + i)} = \frac{x + 2i^2 + xi + 2i}{1 - i^2} = \frac{x - 2 + (x + 2)i}{1 + 1} = \frac{x - 2}{2} + \frac{x + 2}{2}i$$

$$\text{Módulo} = \sqrt{\left(\frac{x - 2}{2}\right)^2 + \left(\frac{x + 2}{2}\right)^2} = 2 \rightarrow \sqrt{\frac{x^2 + 4}{2}} = 2 \rightarrow \frac{x^2 + 4}{2} = 4 \rightarrow$$

$$\rightarrow x^2 + 4 = 8 \rightarrow x^2 = 4 \begin{cases} x_1 = 2 \\ x_2 = -2 \end{cases}$$

Hay dos *soluciones:* $x_1 = 2, \; x_2 = -2$

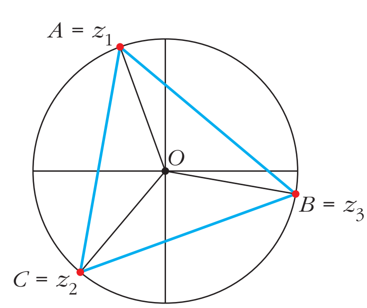
**6.** **Calcula o lado do triángulo cuxos vértices son os afixos das raíces cúbicas de** $4\sqrt{3} - 4i$**.**

$$z = \sqrt[3]{4\sqrt{3} - 4i}$$

Expresamos $4\sqrt{3} - 4i$ en forma polar:

$$\left.\begin{aligned} r &= \sqrt{(4\sqrt{3})^2 + (-4)^2} = 8 \\ tg\, \alpha &= -\frac{1}{\sqrt{3}} \rightarrow \alpha = 330° \end{aligned}\right\} \; 4\sqrt{3} - 4i = 8_{330°}$$

$$z = \sqrt[3]{8_{330°}} = \sqrt[3]{8}_{\frac{330° + 360°k}{3}} \begin{cases} z_1 = 2_{110°} \\ z_2 = 2_{230°} \\ z_3 = 2_{350°} \end{cases}$$



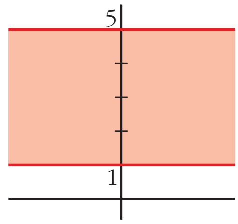

En el triángulo $AOB$ conocemos dos lados, $\overline{OA} = \overline{OB} = 2$, y el ángulo comprendido, 120°. Aplicando el teorema del coseno, obtenemos el lado del triángulo, $\overline{AB}$:

$$\overline{AB}^2 = 2^2 + 2^2 - 2 \cdot 2 \cdot 2 \cdot cos\ 120° = 12 \rightarrow \overline{AB} = \sqrt{12} = 2\sqrt{3} \text{ u}$$

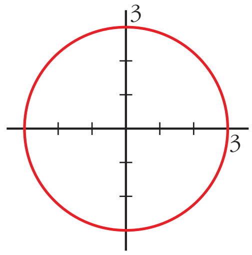
**7.** **Representa gráficamente.**

**a)** $1 \le Im\, z \le 5$ **b)** $|z| = 3$ **c)** $z + \bar{z} = -4$

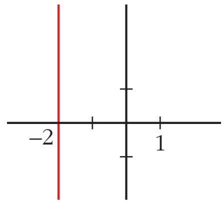
a)




b)




c) $a + bi + a - bi = -4 \rightarrow 2a = -4 \rightarrow a = -2$




**8.** **Calcula dous números complexos tales que o seu cociente sexa $2_{150°}$ e o seu produto $18_{90°}$.**

$\frac{r_\alpha}{s_\beta} = 2_{150°} \rightarrow \frac{r}{s} = 2; \; \alpha - \beta = 150°$

$r_\alpha \cdot s_\beta = 18_{90°} \rightarrow r \cdot s = 18; \; \alpha + \beta = 90°$

Resolvemos los sistemas:

$\begin{cases} r/s = 2 \\ r \cdot s = 18 \end{cases} \quad \begin{cases} \alpha - \beta = 150° \\ \alpha + \beta = 90° \end{cases}$

Obtenemos:

$\begin{cases} r = 6 \\ s = 3 \end{cases} \quad \begin{cases} \alpha = 120° \\ \beta = -30° = 330° \end{cases}$

Los números son $6_{120°}$ y $3_{330°}$. Otra posible solución es: $6_{300°}$ y $3_{150°}$.

**9.** **Demostra que $|z \cdot \bar{z}| = |z|^2$.**

$$\left.\begin{array}{l} z = a + bi \\ \bar{z} = a - bi \end{array}\right\} z \cdot \bar{z} = (a + bi)(a - bi) = a^2 - b^2 i^2 = a^2 + b^2$$

$$|z| = \sqrt{a^2 + b^2} \rightarrow \left.\begin{array}{l} |z \cdot \bar{z}| = \sqrt{(a^2 + b^2)^2} = a^2 + b^2 \\ |z|^2 = \left(\sqrt{a^2 + b^2}\right)^2 = a^2 + b^2 \end{array}\right\} |z \cdot \bar{z}| = |z|^2$$

**10.** **Calcula *cos* 120° e *sen* 120° a partir do produto $1_{90°} \cdot 1_{30°}$.**

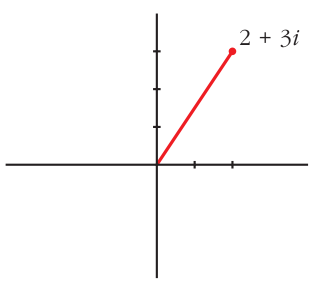
$$1_{90°} \cdot 1_{30°} = 1(\cos 90° + i \operatorname{sen} 90°) \cdot 1(\cos 30° + i \operatorname{sen} 30°) =$$

$$= i \cdot \left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$1_{90°} \cdot 1_{30°} = 1_{120°} = 1(\cos 120° + i \operatorname{sen} 120°) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \rightarrow$$

$$\rightarrow \cos 120° = -\frac{1}{2}; \ \operatorname{sen} 120° = \frac{\sqrt{3}}{2}$$

**11.** **Calcula o número complexo $z$ que se obtén ao transformar o complexo $2 + 3i$ mediante un xiro de 30º con centro na orixe.**




Multipicamos por $1_{30°} = 1(\cos 30° + i \operatorname{sen} 30°)$.

$$z = (2 + 3i) \cdot 1_{30°} = (2 + 3i)\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$z = \sqrt{3} + \frac{3}{2}i^2 + i + \frac{3\sqrt{3}}{2}i$$

$$z = \frac{2\sqrt{3} - 3}{2} + \frac{2 + 3\sqrt{3}}{2}i$$